BOOKS BY HARRY PARKER

Simplified Design of Structural Timber
Simplified Design of Structural Steel
Simplified Design of Reinforced Concrete
Simplified Design of Roof Trusses for Architects and Builders
Simplified Engineering for Architects and Builders
(*Second Edition*)

Kidder-Parker Architects' and Builders' Handbook
By the late Frank E. Kidder and Harry Parker
(*Eighteenth Edition*)

Materials and Methods of Architectural Construction
By Charles Merrick Gay and Harry Parker
(*Second Edition*)

Simplified Design of Roof Trusses
for
Architects and Builders

BY

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NEW YORK
JOHN WILEY & SONS, Inc.
London: CHAPMAN & HALL, Limited

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SEVENTH PRINTING, MARCH, 1949

Printed in U. S. A.

PREFACE

The volume *Simplified Engineering for Architects and Builders* was published two years ago. The primary purpose was to present in a concise manner the basic principles and methods underlying the design of structural members used in building construction. The author had in mind the great number of young men who lacked technical training and who were anxious to acquaint themselves with the necessary knowledge to compute the sizes of various structural elements. It is gratifying that that book has proved successful and has served the purpose for which it was written.

Included in the book was a rather short chapter on roof trusses, the nature of the subject preventing an adequate treatment. Among the letters of comment have been requests for a book written in the same manner and presenting the essential principles and methods of designing the most common types of roof trusses.

The present volume is prepared for those who have had no preliminary training in the design of roof trusses. The stresses in truss members are developed by means of graphical analysis. Beginning with simple systems of forces, the theory of graphic statics is evolved showing the application of the fundamental principles in constructing stress diagrams for various types of trusses and loading.

The chapter entitled "Miscellaneous Roof Trusses" treats of more advanced trusses requiring special solutions. It may be omitted if desired since it is not essential in the study of design of the trusses commonly used. It is presented for reference purposes or for those who wish further investigation.

In addition to graphical analyses, tables of coefficients of stresses for the most usual trusses are included, with examples to illustrate their application. Throughout the book appear the solutions of numerous practical examples supplemented by problems to be solved by the student.

Examples of the design of both timber and steel trusses are presented. All necessary tables of unit stresses, safe loads, properties of sections, etc., are given as means of ready reference. For general purposes no additional handbooks are required.

Throughout the book will be found basic formulas or principles pertaining to mechanics. For students unfamiliar with them, or for those who need to refresh their memories, specific references are made to articles in *Simplified Engineering for Architects and Builders*. Naturally such material may be found in numerous books, this particular book being referred to merely because it is often advantageous to have a direct reference.

The present volume has been prepared in a manner to make it adaptable to home study as well as for use in the classroom. Students preparing for state board examinations should find it adequate for their needs. The design of many roof trusses requires the services of an experienced engineer, but it is the hope of the author that this book will familiarize the student with the basic principles of design and give him a thorough preparation for advanced study.

HARRY PARKER

HIGH HOLLOW, SOUTHAMPTON, PA.
December, 1940

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PART I

PRINCIPLES OF GRAPHIC STATICS

CHAPTER 1

CONCURRENT FORCES

1. **Introduction.** The design of roof trusses consists first in computing the loads on the various joints. After the loads have been established, the character and magnitude of the stresses in the members of the truss are determined. Following this the designer selects the actual sizes of the members to be used and details the connections or joints.

There are several methods that may be used to determine the stresses in truss members but possibly the most convenient method is by means of the principles of graphic statics. *Statics* is the science which deals with forces in equilibrium. *Graphic statics* is nothing more than the solution of statical problems by the use of geometrical diagrams.

A *truss* is a framed structure so designed that the stresses in the members are capable of holding the external forces in equilibrium. Since for our purposes equilibrium consists in forces whose combined effect produces no motion or change in state of rest, all problems concerning roof trusses have as a fundamental datum the condition of equilibrium. This, of course, does not include deformation in members due to expansion and contraction and the elasticity of the materials used.

2. **Force.** A *force* may be defined as an action which produces motion, pressure or tension. There are three properties or elements of a force, namely

1. Magnitude
2. Direction
3. Line of action

In order to completely determine a force it is essential that we establish each of these three elements. The graphical solution of problems dealing with forces is based upon representing a force by a drawn line.

3. **Magnitude.** The *magnitude* of a force is its number of units of energy. Forces are usually given in units of pounds. When we represent a force by a line, the magnitude of the force is expressed by the *length* of the line drawn to a convenient scale. For instance, if a force of 3,000# is to be indicated at a scale of 1″ equals 1,000#, the line will be 3″ in length. Frequently forces are given in units of *kips*, a kip representing 1,000#.

4. **Direction.** The *direction* of a force is the direction in which a force tends to move an object. It may be upward, downward, to the right or left, etc. When a force is represented graphically, its direction is shown by an arrow placed on the line.

5. **Line of Action.** The *line of action* of a force is the path along which it acts. It is represented graphically by the position at which the line is drawn. When a force acts upon a body, the point of contact is known as the *point of application*. Obviously the point of application is a point in the line of action of the force.

6. **Equilibrium.** A system of forces is said to be in equilibrium when their combined effect produces no motion.

7. **Resultant.** The *resultant* of two or more forces is the simplest system (usually a single force) which has the same effect as the given forces. One of the common and useful problems in the science of graphic statics is to find the resultant of a given system of forces, in other words to find, if possible, a single force equivalent to the given system.

8. **Equilibrant.** The simplest system (usually a single force) that will hold a given system in equilibrium is called the *equilibrant* of the system. The equilibrant of a system of forces has the same magnitude and line of action as the resultant, but acts in the opposite direction.

9. **Component.** If a system of forces has one force as a resultant, any force of the given system may be called a *component* of the resultant.

10. **Concurrent Forces.** A system of forces whose lines of action meet in a common point is called a *concurrent system*. If the lines of

action do not have a point in common the forces are said to be *non-concurrent*.

11. **Coplanar Forces.** Forces whose lines of action lie in one plane are called *coplanar forces*. Unless otherwise mentioned, all forces given in the following problems are coplanar.

12. **Mechanical Couple.** A *mechanical couple* consists of two parallel forces, equal in magnitude, opposite in direction and having different lines of action. A mechanical couple tends to cause rotation and can be held in equilibrium only by the addition of another mechanical couple which tends to cause rotation in the opposite direction.

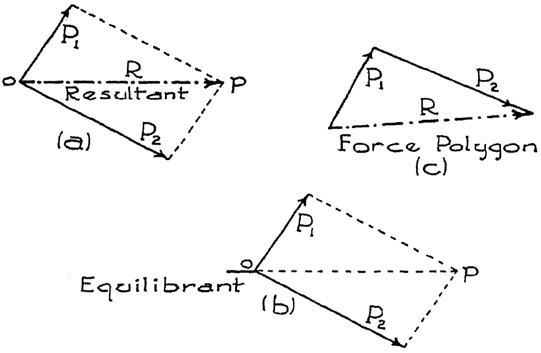


FIG. 1

13. **Resultant of Two Concurrent Forces.** The simplest method of finding the resultant of two concurrent forces is by use of the principle known as the *parallelogram of forces*.

Let it be required to find the resultant of the two forces P_1 and P_2 which have the point o in common, Fig. 1 (a). The magnitudes of the forces are represented by the lengths of the lines and the arrows indicate their directions. From the extremity of P_1 draw a line parallel to P_2 and from the extremity of P_2 draw a line parallel to P_1. These two construction lines form a parallelogram, and the diagonal R passing through the point o is the resultant of the two forces P_1 and P_2, in magnitude and direction. The magnitude is found by measuring the length of line R at the same scale used in laying off P_1 and P_2.

The forces P_1 and P_2 are not in equilibrium, and their combined effect would tend to move a particle from point o to point p. Note that the resultant R acting alone would produce the same effect. If the problem were to find a force that would hold P_1 and P_2 in equilibrium, we could proceed as follows: Complete the parallelogram of forces for P_1 and P_2, Fig. 1 (*b*). The diagonal from o to p represents the resultant, but the equilibrant is a force opposite in direction, that is, toward the left, equal in magnitude and having the same line of action. The equilibrant is indicated by force E.

14. **Resultant of More than Two Concurrent Forces.** The resultant of more than two concurrent forces may be found by the principle of the parallelogram of forces, as explained in Art. 13, by taking the forces in pairs.

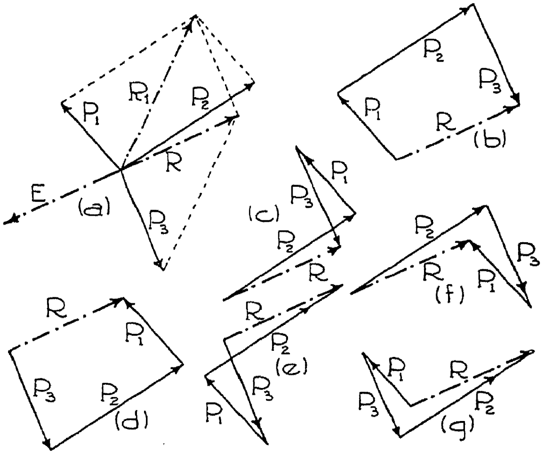


FIG. 2

Suppose we wish to find the resultant of the concurrent forces P_1, P_2 and P_3, Fig. 2 (*a*). First find, by means of the parallelogram of forces, the resultant of P_1 and P_2. This is R_1. Since this resultant produces the same effect as P_1 and P_2 acting together, we may consider that P_1 and P_2 are omitted and R_1, their resultant, is substituted in their place. We now have remaining the forces R_1 and P_3. Next we find, by constructing the parallelogram, the resultant of R_1 and P_3; this is R, and R, consequently, is the resultant of the original system P_1, P_2 and P_3. The equilibrant of the original system is equal to R in magnitude, acts in the opposite direction and has the same line of action. It is indicated by force E.

15. **Force Polygon.** The magnitude and direction of the resultant of a system of forces may also be found by constructing a figure known as a force polygon. To draw a force polygon proceed as follows:

Draw a succession of lines, parallel to the separate forces, equal to them respectively in magnitude and having the same directions, each succeeding line beginning where the preceding line ends. The line required to complete the polygon, *drawn from the starting point*, represents the resultant in magnitude and direction.

If the system of forces is concurrent, the resultant of the system will pass through their point in common. If, however, the system is non-concurrent, the line of action may be found by drawing a funicular polygon as will be explained in Art. 22.

The force polygon for the two forces P_1 and P_2, Figure 1 (*a*) is shown in Fig. 1 (*c*). For the three forces P_1, P_2 and P_3, Fig. 2 (*a*), the force polygons are shown in Figs. 2 (*b*), (*c*), (*d*), (*e*), (*f*) and (*g*). Notice that the forces may be drawn in any sequence; the resultant is the same in each instance. It should be remembered that the force polygon gives the resultant in magnitude and direction only. In these two problems the systems are concurrent and therefore the line of action of the resultant passes through the point common to the system.

A system of concurrent forces is in equilibrium if their force polygon closes. In both of these given systems it was found that the force polygon did not close and therefore the forces are not in equilibrium. The line required to close the polygon is the resultant. Remember, the *direction* of the resultant is found by the direction of the closing line of the polygon *drawn from the starting point.*

Let it be required to find the resultant of the system of five concurrent forces shown in Fig. 3 (*a*). Beginning with P_1, an upward force, construct the force polygon, Fig. 3 (*b*). In this problem it is found that the last force, P_5 closes the polygon. Since the forces are concurrent and the force polygon closes, the system is in equilibrium and the resultant is zero.

Whereas the forces in Fig. 3 (*a*) all act *away* from the common point, the system of concurrent forces shown in Fig. 3 (*c*) acts both toward and away from the point common to the system. This, however, presents no difficulty. Construct the force polygon, Fig. 3 (*d*), taking the forces in any sequence. It is found that the force polygon closes and, as the forces are concurrent, the system is in equilibrium, the resultant being zero.

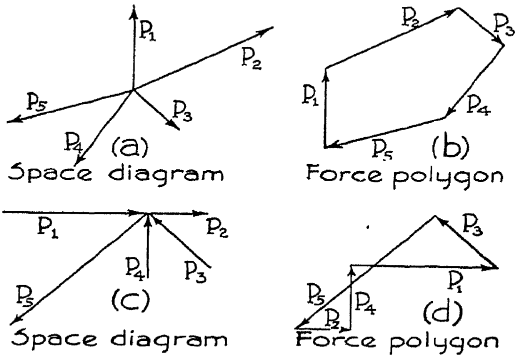


FIG. 3

16. **Resultant of Two Unequal Parallel Forces Having the Same Direction.** The pairs of parallel forces shown in Fig. 4 are of course not concurrent, but by the addition of another pair of forces an equivalent concurrent system may be found. The procedure is as follows:

Forces P_1 and P_2, Fig. 4 (*a*), are two parallel unequal forces having the same direction and different lines of action. Let it be required to find their resultant. By drawing the force polygon, Fig. 4 (*b*), we find the resultant to be an upward force parallel to the forces of the given system and having a magnitude equal to their sum. However, we do not know the line of action of the resultant with respect to the given system shown in the space diagram, Fig. 4 (*a*).

To find the line of action of the resultant, add the two forces A and B as shown. These are equal parallel forces having the same line of action, their resultant being zero. Hence we have not disturbed the original system P_1 and P_2. Next find the resultant of P_1 and A by means of the parallelogram of forces and call it R_1. Likewise find R_2, the resultant of P_2 and B. The system has now been resolved into forces R_1 and R_2, two forces which are not parallel and whose lines of action have the point o in common. *The resultant of two concurrent forces passes through the point common to both.* Since the resultant of R_1 and R_2 is equal to the resultant of P_1 and P_2, R, the resultant of the given system, will pass through point o, the three elements of the force R having now been determined.

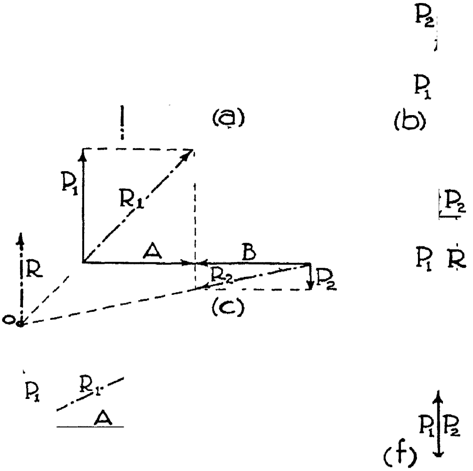


FIG. 4

17. Resultant of Two Unequal Parallel Forces Having Opposite Directions. Forces P_1 and P_2 represent two unequal parallel forces, opposite in direction and having different lines of action, Fig. 4 (c). To find the resultant of this system of forces proceed in exactly the same manner as described in Art. 16. The point of intersection of the lines of action of R_1 and R_2 is point o and the resultant of P_1 and P_2 must pass through this point. The force polygon for forces P_1 and P_2 is shown at Fig. 4 (d), R being parallel to P_1 and P_2 and having a magnitude equal to their difference, the direction being that of the greater force. The force polygon has given the direction and magnitude of R, and its line of action will be through point o.

18. **Resultant of Two Equal Parallel Forces Having Opposite Directions.** To find the resultant of two equal parallel forces, opposite in direction and having different lines of action, apply the method given for the two preceding cases. The original system of forces P_1 and P_2 is readily resolved into R_1 and R_2, Fig. 4 (*e*). However, unlike the former cases, R_1 and R_2 are parallel and of course have no point in common. Hence we have, instead of the original system, another pair of equal parallel forces, opposite in direction and with different lines of action, such a system constituting a mechanical couple. See Art. 12. It should be noted that the force polygon, Fig. 4 (*f*), closes. This, however, does not mean that the system is in equilibrium since the forces are not concurrent. The resultant of two forces constituting a mechanical couple cannot be represented by a single force. The system tends to cause rotation rather than motion of translation. To hold a mechanical couple in equilibrium another mechanical couple must be added.

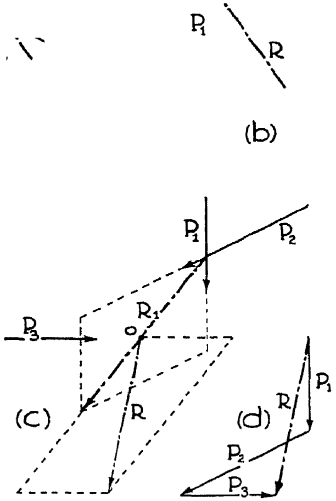


FIG. 5

19. **Forces Continuous on Lines of Action.** In the solution of problems dealing with forces, the forces may be moved to any convenient position *on their lines of action.* The forces P_1 and P_2 in Fig. 5 (*a*) have their lines of action intersecting at point *o*, therefore the resultant of P_1 and P_2 will pass through this point. To find the resultant, move P_2 upward to the left and also move P_1 downward as indicated by the dotted lines. The parallelogram of forces may then be drawn and the resultant, R, found. Fig. 5 (*b*) is the force polygon and shows the resultant of these two forces in magnitude and direction; the resultant, of course, will pass through point *o*.

Three non-concurrent forces, P_1 and P_2 and P_3 are shown in Fig. 5 (*c*). Their force polygon, Fig. 5 (*d*), gives the resultant in magnitude and direction but its line of action with respect to P_1,

P_2 and P_3 is unknown. By the methods previously used their resultant is completely determined by taking the forces by pairs. Moving the forces P_1 and P_2 on their lines of action, their resultant, R_1, is found by the parallelogram of forces. The given system now has been altered to an equivalent system consisting of P_3 and R_1. By transferring these two forces along their lines of action and constructing the parallelogram of forces, R, the resultant of the given system P_1, and P_2 and P_3, is found. It passes through point o, the intersection of R_1 and P_3. The resultant is now completely determined; notice that its magnitude and direction are the same as found in the force polygon, Fig. 5 (*d*).

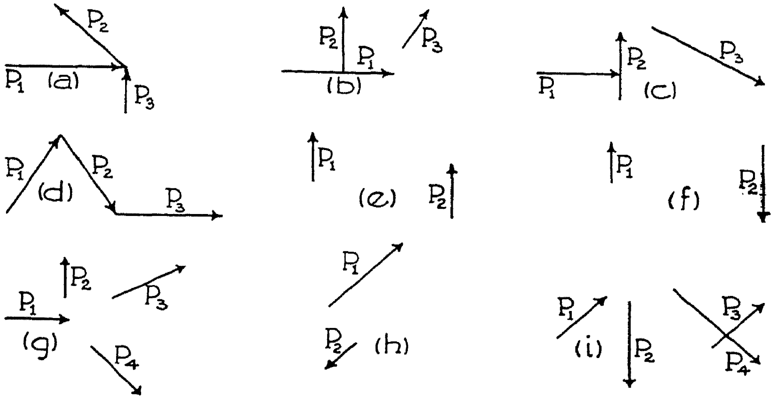


FIG. 6

PROBLEMS 19-A-B-C-D-E-F-G-H-I. Find the resultants of the systems of forces shown in Fig. 6.

CHAPTER 2

NON-CONCURRENT FORCES

20. Notation. In the preceding diagrams the forces have been identified as P_1, P_2, P_3, etc. However, a more convenient system of notation will be used in many of the problems which follow. The method consists in placing a letter in each space that occurs between the forces or their lines of action, each force being identified by the two letters that appear in the adjacent spaces. For instance, Fig. 7 (*a*) represents five concurrent forces. The common point is identified as *BCGFE* and the forces are *BC*, *CG*, *GF*, *FE* and *EB*. Note particularly that these forces have been read in a *clockwise* manner. If a counter-clockwise reading were employed the forces would be *BE, EF, FG, GC* and *CB*. Unless so noted, *all forces in this book will be read in a clockwise manner.* Although no direction is indicated for reading the system of forces in Fig. 7 (*a*), they are read in a clockwise direction, and in the trusses shown later the same method of identifying the forces is employed regardless of the character of the stress. By character of stress is meant compression or tension.

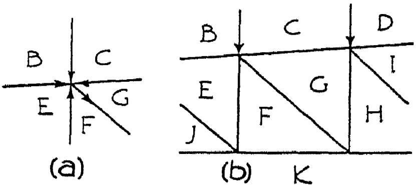


FIG. 7

A portion of a truss is shown in Fig. 7 (*b*). Reading the members in a clockwise fashion about the joint *BCGFE*, the upper chord between the two joints is read *CG*. With respect to the joint *CDIHG*, the same member is read *GC*. In the same manner, the diagonal member is read *GF* with respect to the joint *BCGFE* but about the joint *FGHK* this diagonal member is read *FG*. It is of particular importance that this method of identifying members or forces is clearly understood. A clockwise direction has been used and this is the direction that will be followed in the succeeding problems. Remember that if a member extends from one joint to another it is necessary to have in mind which end is considered before identifying it. We use the expression "with reference to joint . . . ," as, for instance, the member *GH* with reference to the joint *FGHK*, or member *HG* with reference to joint *CDIHG*.

21. **Stresses in Members.** In order to identify truss members and their stresses, it is important that some consistent method of notation be adopted. As previously stated, the forces or truss members will be identified by capital letters placed on each *side* of the member. When, however, a force polygon is drawn showing the magnitude of the stresses in the members, corresponding lower-case letters are placed at the *extremities* of the lines, the length of the lines representing the magnitude of the stresses in the members.

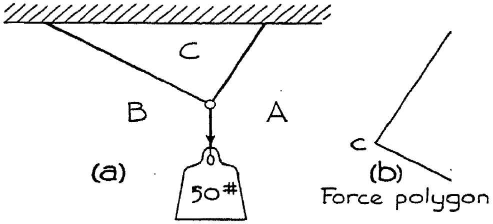


FIG. 8

The following problem illustrates how readily the magnitude of stresses in members may be found by applying the graphical methods we have previously discussed.

Consider, for example, two rods *BC* and *CA* hung from a ceiling, Fig. 8 (*a*). These rods meet at joint *CAB* from which point a load of 50# is suspended on rod *AB*.

A *stress* in a member is the internal resistance which balances an external force. Then, since the member *AB* supports a force of 50#, the stress in *AB* must be 50#. Suppose we are asked to find the magnitude of the stresses in the members *BC* and *CA*.

Fundamentally, we have in this problem three concurrent forces in equilibrium. The forces are *AB*, *BC* and *CA*. Of these three forces only AB is known completely. We know the lines of action of BC and CA but not their magnitudes. In Art. 15 we learned that if a system of concurrent forces is in equilibrium their force polygon must close. This is the key to our solution.

First draw ab, a downward force of 50#, to some convenient scale, as for instance 1″ = 50#, Fig. 8 (b). This will be one side of the force polygon. The next force in order is BC, and from point b draw a line parallel to BC. The point c will be at some point on this line. Next consider the member CA and draw a line through point a parallel to the member CA. The point c will occur somewhere on this line and, since c is at some point on the line through b parallel to BC their point of intersection will establish the point c. This then, completes the force polygon for the three forces AB, BC and CA. To find the magnitude of the stresses in the members BC and CA we merely read the lengths of the lines in the force polygon corresponding to the members, using the same scale used in laying off the given force AB. It is found that the stress in BC is 25# and the stress in CA is 43.3#. How simple it is!

The accuracy in reading the magnitude of the stresses depends upon the accuracy with which the force polygon is drawn. Of course the larger the force polygon, the more accurately may the stresses be read.

Attention is called to the fact that the length of the members in the space or truss diagram bears no direct relation to the magnitude of their stresses. The stresses, however, are directly proportional to the length of the corresponding sides of the force polygon. Since the force polygon determines the stresses within the members, the combined force polygons showing stresses in frames is called a *stress diagram.*

22. **Funicular Polygon.** *If a system of forces is in equilibrium, both the force polygon and funicular polygon will close.* This is a general rule and applies to any system regardless of the number of forces, whether they be concurrent or non-concurrent.

Consider the system of forces, AB, BC and CA shown in Fig. 9 (a). If these three forces constitute a system in equilibrium, both the force polygon and funicular polygon will close. Let us apply the test.

The force polygon abc, Fig. 9 (b), is first drawn. This polygon closes. Thus far it appears that the system may be in equilibrium.

If the force polygon had not closed there would be no need to continue further; the system would not be in equilibrium.

Now let us construct the *funicular polygon.* To do this we begin by selecting any point in connection with the force polygon. This point is called the *pole* and is marked *o*, Fig. 9 (*b*). From this point draw the lines *oa*, *ob* and *oc*. These lines are known as *rays* and represent the stresses in an imaginary frame connecting the three forces *AB*, *BC* and *CA* shown in Fig. 9 (*a*). Our next step is to construct this imaginary frame known as the *funicular or equilibrium polygon.*

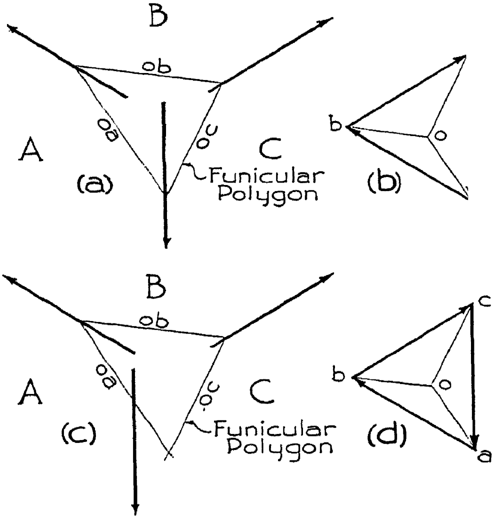


FIG. 9

It is seen in Fig. 9 (*b*) that the force *ab* is held in equilibrium by the rays *oa* and *ob*. Therefore, in Fig. 9 (*a*) select any point on the line of action of the force *AB* and draw lines *oa* and *ob* parallel respectively to rays *oa* and *ob*. These lines, called *strings*, are continued until they intersect the adjacent forces. In the force polygon, Fig. 9 (*b*), force *bc* is held in equilibrium by rays *ob* and *oc*, hence in the space diagram, Fig. 9 (*a*), *BC*, *ob* and *oc* must have a point in common. Consequently, from the point where *ob* intersects force *BC*, draw the string *oc* parallel to ray *oc*. We now observe in Fig.

9 (*a*) that strings *oa* and *oc* intersect on the line of action of force *CA*, that is to say, *the funicular polygon closes,* and, since the force polygon also closes, the given system is in equilibrium.

If a different point had been selected as a pole, a different funicular polygon would have resulted. However, the last two rays drawn would have intersected on the remaining force, showing that the funicular polygon closed and proving that the system is in equilibrium. If we continue the lines of action of the forces *AB*, *BC* and *CA*, in Fig. 9 (*a*), we notice that they intersect at a common point so that actually the system is concurrent.

Again consider the three forces *AB*, *BC* and *CA* in Fig. 9 (*c*). Let us see if this system is in equilibrium. By drawing the force polygon, *abc*, Fig. 9 (*d*), we find that it closes. Next select a pole and draw the rays and strings of the funicular polygon as previously described, taking the forces in the same sequence. It is seen, in this instance, that *oa* and *oc*, the sides of the funicular polygon, do *not* intersect on the force *CA*. *The funicular polygon does not close,* hence this system is not in equilibrium.

Notice that if the lines of action of the forces *AB*, *BC* and *CA* are extended in Fig. 9 (*c*), they do not intersect at a common point. If we were required to alter the system to produce equilibrium, we could move the force *CA* to the right so that its line of action passes through the intersection of strings *oa* and *oc*. We would then have a system of forces whose force and funicular polygons close and hence would be in equilibrium. But observe! If *CA* is moved so that its line of action passes through the intersection of strings *oa* and *oc* we alter the system so that forces *AB*, *BC* and *CA* constitute a concurrent system. This illustrates a principle well worth remembering, namely, *three non-parallel forces in equilibrium must meet in a point.*

23. **Funicular Polygon Used as a Test.** Fig. 10 (*a*) consists of a system of three forces *AB*, *BC* and *CA;* let it be required to find whether or not they are in equilibrium. In accordance with the principle stated in Art. 22, we know that equilibrium exists provided both the force polygon and funicular polygon close. Let us construct these polygons. First draw the force polygon, Fig. 10 (*b*). We observe that it closes. If the system had been concurrent it would not be necessary to proceed further, for the closing of the force polygon would have assured us that the forces were in equilib-

rium. But the system is not concurrent; therefore we must test it further by constructing a funicular polygon.

To do this, select any point *o* in the force polygon and draw the rays *oa*, *ob* and *oc*. In the force polygon we find that force *ca* is held in equilibrium by *oa* and *oc*; therefore draw from point *r*, any point in the line of action of *CA*, lines parallel to *oa* and *oc*. *AB* is intersected by line *oa* at *s*, and from this point draw a line *ob* parallel to the ray *ob*. In this problem we find that *oc* and *ob* intersect at point *t* which is *not* on the line of action of the force *BC*. "The polygon does not close," and therefore the system is not in equilibrium. Note that, if the force *BC* is moved to a parallel position whereby its line of action passes through point *t*, the altered system would be concurrent and hence in equilibrium.

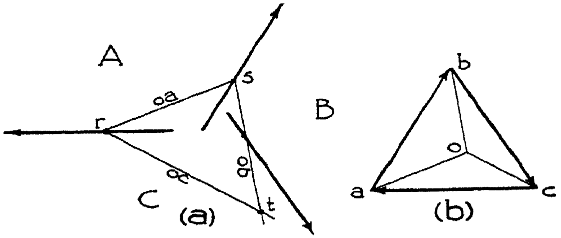


FIG. 10

Actually the given system in this problem is equivalent to a mechanical couple. This is apparent if we consider the forces in pairs. The resultant of *CA* and *AB* is *CB*, found by the force polygon. But the resultant of two concurrent forces passes through the point their lines of action have in common. The resultant of *CA* and *AB* and the remaining force *BC* constitute a mechanical couple, as they are two equal parallel forces acting in opposite directions and having different lines of action.

A system of forces whose force polygon closes, but whose funicular polygon does not close, constitutes a mechanical couple.

24. **Finding the Resultant by Use of the Funicular Polygon.** Let it be required to find the resultant of the system of four forces, *AB*, *BC*, *CD* and *DE* shown in Fig. 11 (*a*). The system is nonconcurrent, and the resultant might be found by use of the parallelogram of forces, taking the forces in pairs. This method, however,

would be unnecessarily long, and the most direct procedure is to construct the funicular polygon. To do this, draw the force polygon, Fig. 11 (*b*). This gives us the resultant *ae* in magnitude and direction but does not tell us its position with respect to the system. The pole is selected (any point such as *o*) and the rays are drawn,

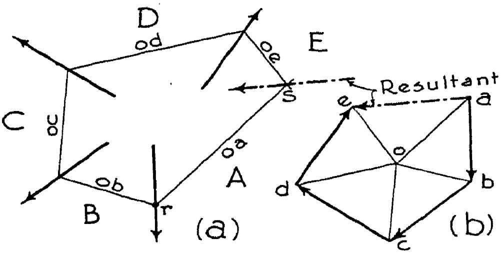


FIG. 11

oa, *ob*, *oc*, *od* and *oe*. Next select some point on the line of action of one of the forces, as point *r* on force *AB*, and draw lines parallel to the rays *oa* and *ob*. Where the line *ob* intersects *BC* draw the line *oc* parallel to the ray *oc*. In a similar manner continue to draw the

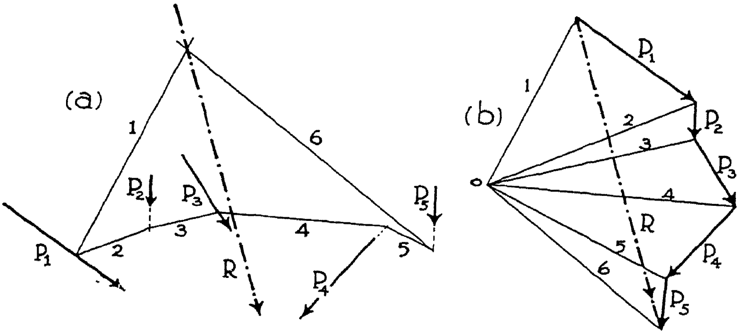


FIG. 12

sides of the funicular polygon until finally we have the two sides *oa* and *oe* which intersect at point *s*. Since, in the force polygon we find that *ae*, the resultant, is held in equilibrium by *oa* and *oe*, the point of intersection, *s*, determines the point through which the resultant will pass, thus determining its line of action.

Another example of finding the resultant of a system of forces by constructing the funicular polygon is shown in Fig. 12. The given system consists of P_1, P_2, P_3, P_4 and P_5, Fig. 12 (*a*). Drawing the force polygon, Fig. 12 (*b*), shows us that the resultant is a downward force marked R. In this example the rays have been identified as 1, 2, 3, 4, 5 and 6. Since the rays 1 and 6 hold R in equilibrium, the sides of the funicular polygon corresponding to these rays intersect at a point through which the resultant will pass and thus determines the line of action of the resultant.

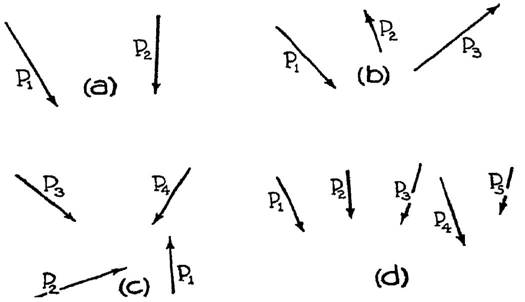


FIG. 13

PROBLEMS 24-A-B-C-D. Find the resultants of the systems of forces shown in Fig. 13 (*a*), (*b*), (*c*) and (*d*) by use of the funicular polygon.

25. **Resultant of Parallel Forces.** A method of finding the resultant of parallel forces was explained in Arts. 16 and 17. A more convenient solution of finding the resultant is by use of the funicular polygon. Consider, for example, the parallel forces AB, BC, CD, DE and EF shown in Fig. 14 (*a*). The force polygon is drawn, Fig.

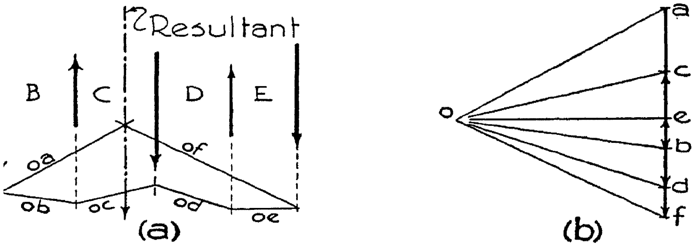


FIG. 14

14 (*b*), and, since the forces are parallel and are both upward and downward, the polygon overlaps, the resultant being AF. Select the pole o, draw the rays and construct the funicular polygon connecting the forces as shown in Fig. 14 (*a*). The intersection of the sides of the funicular polygon oa and of determines the point through which the resultant AF will pass, the magnitude and direction being found by the force polygon.

26. **Reactions Found by Funicular Polygon.** One of the most important uses of the funicular polygon is in finding the reactions of beams and trusses. The reactions are the upward forces under a beam that hold the downward forces in equilibrium. If a beam is symmetrically loaded, of course the reactions are equal, and each has a magnitude equal to one-half the sum of the downward forces or loads. However, if the loading is not symmetrical, the reactions are not necessarily equal and although we know their directions to be upward their magnitudes are unknown. They may be solved mathematically by the use of the principle of moments but their magnitudes may also be found by constructing the funicular polygon. Frequently the latter method is more advantageous.

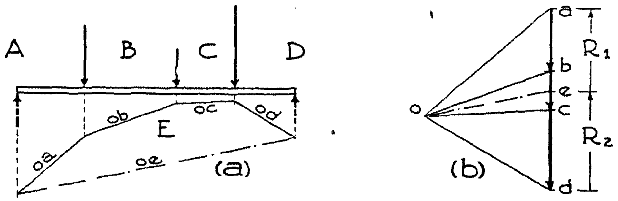


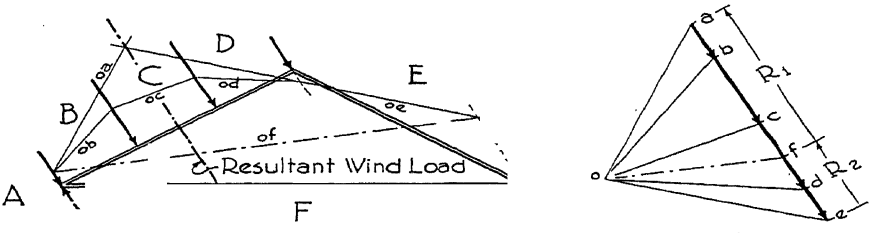
FIG. 15

Fig. 15 (*a*) represents three parallel forces AB, BC and CD exerting pressure on a beam. The two upward forces EA and DE, known respectively as R_1 and R_2, are the two reactions that hold in equilibrium the three downward forces. We know the lines of action of these reactions and also their directions, but their magnitudes are unknown.

First draw as much of the force polygon as possible, Fig. 15 (*b*). This consists of the downward forces AB, BC and CD. The next force in order is DE, an upward force. Since its magnitude is unknown as yet, the point e cannot be located on the force polygon. Obviously it will be at some point on the load line, for $DE + EA$

must equal $AB + BC + CD$ as the five forces are parallel and in equilibrium, and the force polygon must close. Now select a point *o* and draw the rays *oa*, *ob*, *oc* and *od*.

The next step is to construct the funicular polygon, taking, for instance, some point on the line of action of the force *AB* and drawing the sides *oa* and *ob* parallel to the rays *oa* and *ob*. At the point where *ob* intersects the line of action of *BC* draw *oc* parallel to the ray *oc*. Continuing in the same manner, draw *od*. Now we remember that if a system of forces is in equilibrium both the force polygon and funicular polygon must close. Therefore, from the intersection of the side *oa* and the force *EA* draw the line *oe* to the point of intersection of *od* and *DE*. This is the closing line of the funicular polygon and is called the "closing string." Having determined *oe*, we draw a parallel line from the pole to the force polygon, Fig. 15 (*b*). This line, known as the "closing ray," determines the point *e* on the force polygon, consequently establishing the magnitudes of the reactions, *DE* and *EA*.



(a) (b)

FIG. 16

Fig. 16 (*a*) represents four wind loads, *AB*, *BC*, *CD* and *DE*, which exert pressure on a truss. The truss is shown by the double lines, the web members being omitted. Let us assume the reactions at the supports to be parallel to the direction of the wind. We know then their lines of action and direction but not their magnitudes. By using the method explained in the previous problem these magnitudes are readily found.

First draw the loads *AB*, *BC*, *CD* and *DE*, select the point *o* and draw the rays, Fig. 16 (*b*). Next select some point on the line of action of one of the loads and draw the two sides of the funicular polygon. For instance, take a point on *BC* and draw the lines *ob* and *oc*. The funicular polygon is developed as has been explained and, knowing that the forces are in equilibrium and that the funicular polygon must close, we draw *of*, the closing string. From point *o* in Fig. 16 (*b*) we draw a line parallel to the closing string *of*, and this determines the point *f* in the force polygon. Hence the magnitudes of R_1 and R_2, the two reactions, are established.

In many problems it is convenient to know the position of the resultant of the wind loads. By referring to Fig. 16 (*b*) we see that the resultant is *ae* and that it is held in equilibrium by the rays *oa* and *oe*. Therefore, the intersection of *oa* and *oe* in Fig. 16 (*a*) determines a point through which the resultant must pass.

27. To Find Reactions, Line of Action of One Being Unknown. The following problem frequently occurs in the solution of roof trusses. It consists of finding the reactions or supporting forces, when the line of action of one reaction is known while all that is known of the other reaction is a point in its line of action. The example given here consists of forces on a beam, but the method employed is applicable to forces on a truss as well.

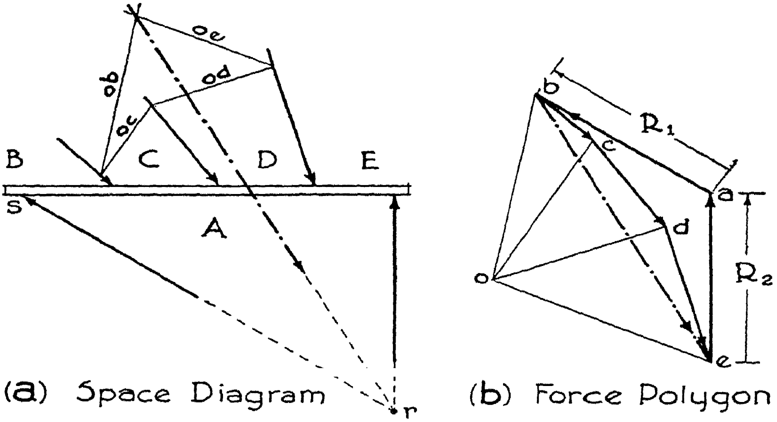


FIG. 17

Fig. 17 (*a*) represents three forces *BC*, *CD* and *DE* acting on a beam. Let it be assumed that the right-hand reaction, *EA*, is vertical and that its magnitude is unknown. Assume also that the line of action of *AB*, the left reaction, is unknown as well as its magnitude. The points of support are, of course, points in the line of action of the two reactions. Let it be required to determine completely *EA* and *AB*, the reactions.

The problem may be simplified if we consider that we have three forces in equilibrium, the resultant of the loads and the two reactions. Here is where we employ the knowledge we have gained concerning resultants. The loads consist of the forces *BC*, *CD* and *DE*. To find their resultant is a simple matter. By drawing the force polygon the resultant is found to be *be*, a downward force to the right. Next select a pole, *o*, and draw the rays. By constructing the funicular polygon the sides *ob* and *oe* intersect at a point through which the resultant *BE* must pass. Now we may forget the individual forces *BC*, *CD* and *DE* and consider in their stead their resultant *BE*. The system now consists of three forces *BE*, *EA* and *AB*. Since by data they are in equilibrium and are not parallel, they must have a point in common. This is readily found; we merely continue the line of action of *BE* until it intersects the vertical force *EA* at *r*. This is the point the three forces have in common. We know that *AB* must pass through *s*, the point of support, and therefore by drawing a line through *r* and *s* we determine the line of action of *AB*.

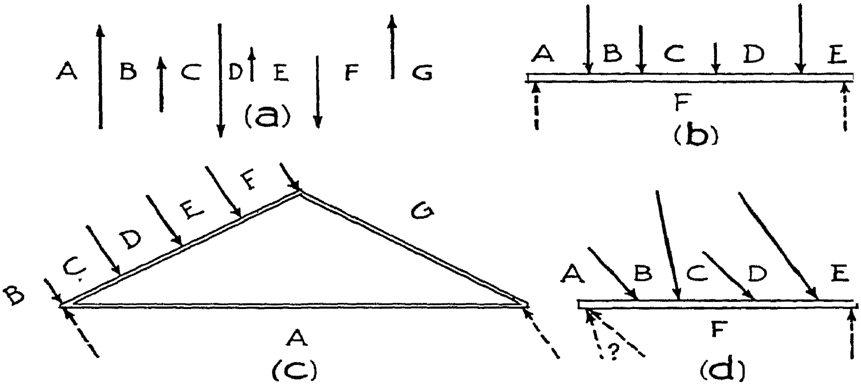


FIG. 18

Returning to the force polygon, we have already drawn the three forces *BC*, *CD* and *DE*. The next force in order is *EA*. Therefore, through the point e draw a line parallel to EA; this, of course, is vertical, and the point a will occur somewhere on this line. The next force is AB, and through the point b draw a line parallel to AB. The point a will be somewhere on this line. Since a is on the line through e parallel to EA and also on the line through b parallel to AB, it must be at their point of intersection. This, then, completes the force polygon and we have completely determined the two reactions R_1 and R_2, their magnitudes being found by measuring their lengths in the force polygon.

PROBLEM 27-A. Find the resultant of the system of forces shown in Fig. 18 (*a*).

PROBLEM 27-B. Find the reactions of the beam for the loads indicated in Fig. 18 (*b*).

PROBLEM 27-C. Five wind loads are shown acting on the frame in Fig. 18 (*c*). Find the reactions, assuming them to be parallel to the direction of the wind.

PROBLEM 27-D. Fig. 18 (*d*) represents four forces acting on a beam. Let it be assumed that EF is vertical and that the line of action of FA is unknown. Find the reactions.

CHAPTER 3

CENTROIDS—CENTER OF GRAVITY

28. **Centroids.** In problems dealing with areas it is often necessary to locate their centroids. The *centroid* of a plane surface is the point which corresponds to the center of gravity of an extremely thin homogeneous plate of the same shape and area. For regular areas the position of the centroid may readily be found by geometrical construction, while for irregular areas the centroid may be located by making use of the funicular polygon. Such problems may, of course, be solved by mathematics, using the principle of moments, but frequently the graphical method saves time, the accuracy of the result depending upon the accuracy employed in making the drawing.

29. **Centroid of a Parallelogram.** From the principles evolved in geometry, the centroid of a parallelogram is readily determined.

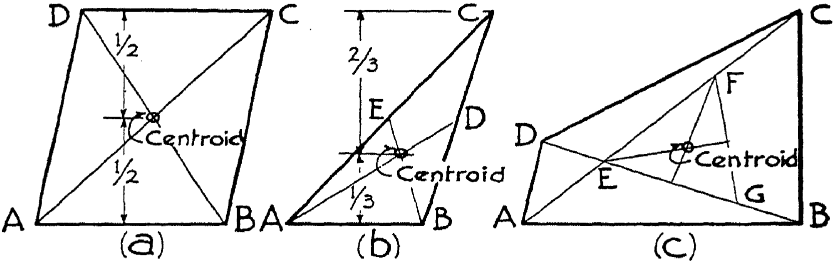


Fig. 19

ABCD, Fig. 19 (*a*), represents a parallelogram. To find the centroid, draw the diagonals *AC* and *BD*. Their point of intersection determines the centroid of the area. This point is always on a line which is halfway between the two opposite sides.

30. **Centroid of a Triangle.** The centroid of a triangle lies on a line drawn from any vertex to the middle point of the opposite side. Let *ABC*, Fig. 19 (*b*), represent any triangle. To find the centroid, draw a line from *A* to the point *D* which is midway between

points *B* and *C*. The centroid lies on this line *AD*. Next construct a line from *B* to *E*, the midpoint of line *AC*. Since the centroid lies on this line *BE* as well as on the line *AD*, their intersection locates the position of the centroid of the triangle *ABC*. It is of convenience to remember that the centroid of a triangle lies at a point one-third the altitude taken from the base.

31. **Centroid of a Quadrilateral.** To find the centroid of a quadrilateral first draw the two diagonals. From the ends of the diagonals farthest from the intersection, lay off distances equal to the shorter segments. These two points and the point of intersection of the diagonals form a triangle, the centroid of which is the centroid of the quadrilateral. For instance, in the quadrilateral *ABCD*, Fig. 19 (*c*), draw the two diagonals *AC* and *BD* which intersect at point *E*. On the line *AC* lay off *CF* equal to *AE*. Similarly, on *BD* lay off *BG* equal in length to *DE*. By the method explained in Art. 30, find the centroid of the triangle *EFG*. This centroid is the centroid of the quadrilateral *ABCD*.

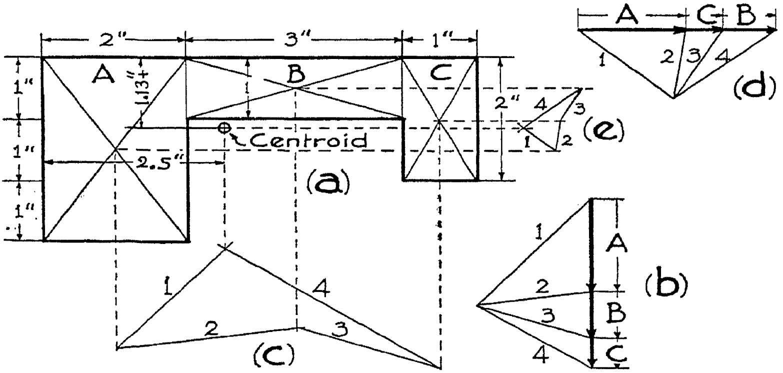


FIG. 20

32. **Centroid of an Irregular Area.** To find the centroid of an irregular area, the given figure is first divided into a number of elementary areas, the centroids of which are readily found. These areas may be considered as individual parallel forces which are proportional in magnitude to the respective areas. For this system of parallel forces a resultant is found which passes through the centroid of the given area.

Let it be required to find the position of the centroid of the irregular area shown in Fig. 20 (*a*). Since this area is unsymmetrical with respect to both the vertical and horizontal axes, it will be necessary to complete two separate operations in order to locate the centroid.

This area is conveniently divided into three rectangular areas lettered *A*, *B* and *C*, the areas of which are 6in^2, 3in^2 and 2in^2 respectively. Since these three areas are parallelograms, their individual centroids are found by their intersecting diagonals. We may now consider the given section as three separate areas, the centroids of which are known. Or, if the given section were one unit in thickness we might think of the weight of the whole mass as being made up of three separate forces, *A* of six units of weight, *B* of three units and *C* of two units. If we think of these three forces acting downward our problem is to find the position of their resultant. This is a simple matter; first draw the load line consisting of the three forces *A*, *B* and *C*, using any convenient scale and making the forces proportional to the respective areas, namely, 6, 3 and 2, Fig. 20 (*b*). A pole is selected and the rays 1, 2, 3 and 4 are drawn. Next draw the funicular polygon, Fig. 20 (*c*). From the force polygon we see that the resultant of the forces *A*, *B* and *C* is a downward force equal to their sum and is held in equilibrium by the rays 1 and 4. Therefore, the intersection of the sides of the funicular polygon corresponding to rays 1 and 4 determines a point through which the resultant must pass. Consequently, the centroid of the given section will lie at some point on the line of action of this resultant. Thus far we have found the position of the centroid with respect to a vertical axis. By measuring the position of the resultant we find it to be 2.5″ from the left side of the figure.

The section being unsymmetrical, the same process must be followed to find the position of the centroid with respect to a horizontal axis. The lines of action of the three areas, or forces, *A*, *C* and *B*, are drawn horizontally and the force polygon and funicular polygons drawn, Fig. 20 (*d*) and (*e*). The intersection of the sides of the funicular polygon, 1 and 4, determines the line of action of the resultant of *A*, *C* and *B*, and therefore the centroid of the entire section lies on this horizontal line of action. The intersection of this line of action and the vertical line of action of the resultant previously found determines the position of the centroid of the given section. This is a graphical method of locating the centroid.

Regardless of the shape of the area, it may be divided into a number of smaller areas and the position of the centroid found by the principle of constructing the funicular polygon.

33. **Position of Centroid by Moments.** In locating the position of centroids by graphical methods, it is frequently of assistance to verify the results by mathematics. In practice, the two methods are often combined in order to save time. Let us see how closely the results found in the preceding example agree with a mathematical solution.

From the study of mechanics we know that the statical moment of a plane area is the product of the area by the normal distance of the centroid of the area to the axis about which the moment is to be taken.* In addition, we know that the statical moment of the entire area is equal to the sum of the statical moments of its parts with respect to the same axis. This principle enables us to write an equation, the solution of which gives us the position of the centroid. Remember that in considering a moment we must have in mind a point or axis about which the moment is taken.

In this problem let us first take the left-hand side of the area as the axis. The same three areas A, B and C will be considered, the areas being 6in^2, 3in^2 and 2in^2, and the distances of their individual centroids to this axis are $1''$, $3.5''$ and $5.5''$ respectively. The sum of the moments of the parts then will be

$$(6 \times 1) + (3 \times 3.5) + (2 \times 5.5)$$

The entire area is $6 + 3 + 2$ or 11in^2 but the distance of the centroid to the assumed axis is as yet unknown. Let us call it x''. The moment of the entire area therefore will be $11 \times x$, and our equation may be written

$$(6 \times 1) + (3 \times 3.5) + (2 \times 5.5) = 11 \times x$$

and

$$x = 2.5''$$

Note that the distance of the centroid from the left side of the figure found by this method is the same as was found by the graphic method.

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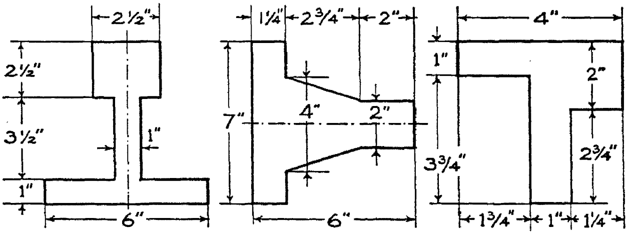
* See Art. 41, *Simplified Engineering for Architects and Builders*, John Wiley & Sons, Inc., New York.

To find the distance of the centroid from the uppermost surface of the area, we write in a similar manner an equation of moments, taking the upper edge as the axis of moments, or

$$(6 \times 1.5) + (3 \times 0.5) + (2 \times 1) = 11 \times y \qquad y = 1.13+''$$

Again we find that the solution by mathematics agrees with the graphic solution.



(a) (b) (c)

FIG. 21

PROBLEMS 33-A-B-C. Find the position of the centroids in Figs. 21 (*a*) (*b*) and (*c*) by graphical methods and check the results by mathematics.

CHAPTER 4

BEAMS, SHEAR AND MOMENT DIAGRAMS

34. **Beams.** Graphical methods may readily be employed in drawing shear and bending-moment diagrams for beams. In certain cases the graphical solution affords a saving of time when compared with the customary mathematical process. This is particularly true for complex loading of beams and girders in which the values of the shear and moment are required at several points along its length. Before attempting the graphical solutions, even though they are surprisingly simple, it is necessary to understand thoroughly the methods used employing mathematics.

Perhaps you may wonder why we wish to draw these shear and moment diagrams. The real purpose, of course, is to design the beam, that is, to select a beam of the proper size to support the given loads safely. To accomplish this we must know the magnitude of the maximum bending moment, and this value cannot be determined until we have first found the point in the beam at which it occurs. Remember, the maximum bending moment occurs at the point where the shear passes through zero, and this point is readily observed by inspecting the shear diagram.

For the first illustration, a simple beam with concentrated loads, both the graphical and mathematical solutions will be given.

35. **Simple Beam with Concentrated Loads.** Let us consider first the simple beam having a span of 20′0″ with three concentrated loads as shown in Fig. 22 (*a*). First draw the beam and loads at some convenient scale. In this instance the forces are lettered so that *AB*, *BC* and *CD* represent the loads, and *DE* and *EA* the reactions. Next draw *ab*, *bc* and *cd* as a portion of the force polygon, Fig. 22 (*b*). In order to determine the magnitude of the reactions, the pole *o* is selected and the rays *oa*, *ob*, *oc* and *od* are drawn. Construct the funicular polygon, Fig. 22 (*d*), and draw the ray *oe* parallel to the closing string *oe*, thus determining the position of *e* and therefore the two reactions *DE* and *EA*. The magnitude of the

reactions may be found by scaling the distances *de* and *ea* in the force polygon, using the same scale as in laying off the loads. We find them to be $DE = 8{,}800\#$ and $EA = 7{,}200\#$.

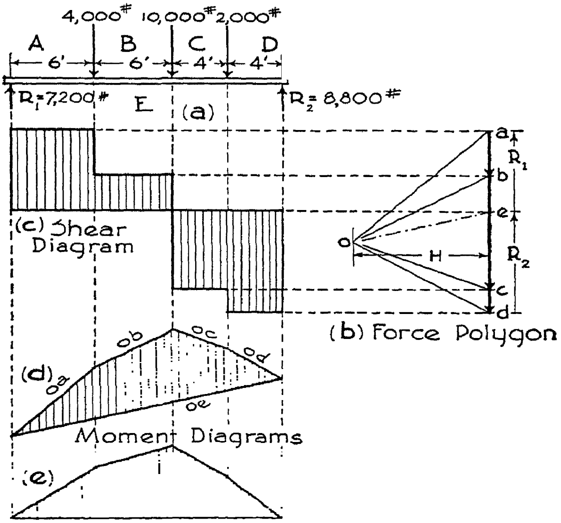


FIG. 22

Let us now compute the two reactions, R_1 and R_2, by mathematics* to see how nearly the results check with the graphical method.

Writing an equation of moments about R_2 as an axis,

$$20\,R_1 = (4{,}000 \times 14) + (10{,}000 \times 8) + (2{,}000 \times 4)$$
$$20\,R_1 = 144{,}000$$
$$R_1 = 7{,}200\#$$

Since

$$R_1 + R_2 = 4{,}000 + 10{,}000 + 2{,}000$$
$$7{,}200 + R_2 = 16{,}000$$

and

$$8{,}800\#$$

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* See Art. 23, *Simplified Engineering for Architects and Builders.*

The value of the vertical shear at any section of a beam is equal to the algebraic sum of the forces on either side of the section. Or we may say, *the value of the vertical shear is equal to the reactions minus the loads to the left of the section.** To draw the shear diagram without the use of mathematics we merely repeat the above rule and make the drawing accordingly. Let us try.

First consider any section between R_1 and the load AB. Since by the above rule we know the shear equals the reactions minus the loads to the left of the section, and as there are no loads to the left, the value of the shear at this section equals the reaction minus zero. Using the letters shown for the forces, the shear here equals EA minus zero, or EA. Hence we merely project *ea* in the force polygon horizontally to the left to the space between EA and AB directly below the beam. Note that *ea* reads upward, a positive quantity. In accordance with the conventional method of representing the shear, the positive values are shown above the base line and the negative or minus values are placed below.

Again, take any section between the loads AB and BC. The value of the shear is EA minus AB, or EB, a positive quantity. This value is likewise projected horizontally from the force polygon.

The value of the shear at any section between BC and CD is EA minus $(AB + BC)$, or EC, a downward or negative quantity. In the same manner the shear at any section between CD and DE, the right reaction, equals $EA - (AB + BC + CD)$, or ED. Projecting *ec* and *ed* to their respective positions below the beam, the shear diagram is now complete. The scale for the shear diagram will be the same scale used in drawing the force polygon.

The values of the shear at these four critical sections in the beam are readily found by mathematics as follows:

The shear at any section between EA and $AB = 7{,}200 - 0 = +\ 7{,}200\#$

The shear at any section between AB and $BC = 7{,}200 - 4{,}000 = +\ 3{,}200\#$

The shear at any section between BC and $CD = 7{,}200 - (4{,}000 + 10{,}000) = -6{,}800\#$

The shear at any section between CD and $DE = 7{,}200 - (4{,}000 + 10{,}000 + 2{,}000) = -8{,}800\#$

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* See Arts. 27 and 28, *Simplified Engineering for Architects and Builders.*

One of the important reasons for drawing the shear diagram is to find the point at which the shear passes through zero. By observing the shear diagram just drawn, this point is seen to be under the force BC. We will expect the bending moment to have a maximum value here.

In order to draw the funicular polygon, Fig. 22 (*d*), it was first necessary to select the point *o*, called the pole. The normal distance from the pole to the force polygon is called the *pole distance*. In this instance, Fig. 22 (*b*), it is the distance marked H. When a funicular polygon is drawn for a beam in order to determine the reactions, it can be shown that at any point in the beam the vertical ordinate of the funicular polygon multiplied by the pole distance determines the value of the bending moment. As the pole distance is constant for parallel loads, *the funicular polygon therefore is actually the bending-moment diagram.* If you are accustomed to drawing moment diagrams, perhaps Fig. 22 (*d*) is not at first recognized. This is because the conventional method is to compute the values by mathematics: draw a horizontal base line beneath the beam and plot the values vertically above and below this line, the positive moments above and the negative moments below. If it is desirable to do so, the familiar diagram is readily drawn, making use of the funicular polygon. This is shown at Fig. 22 (*e*). A horizontal line is drawn directly beneath the funicular polygon, and the *vertical* distances found in the funicular polygon are plotted with respect to this horizontal line. Either diagram may be used as the bending-moment diagram, but remember, the value of the moment at any section in the beam is the corresponding *vertical* distance, or ordinate, of the bending-moment diagram.

Naturally, you have been wondering about the scale to use in reading the moment diagram. It is obvious that, if the pole distance H had been shorter, the vertical distances in the funicular polygon would have been greater. This means that a different scale would have to be used in reading the values, for the bending moments at various sections of the beam are, of course, not dependent on the pole distance.

The scale used for reading the moment diagram depends upon the scale used in drawing the beam diagram and also the pole distance H. The scale of the beam diagram for Fig. 22 (*a*) was $1'' =$

8′0″; the force polygon, Fig. 22 (*b*), was drawn at a scale of 1″ = 8,000# and the pole distance H was 10,000#, hence the moment diagram is read at a scale of 1″ = 80,000′#. Since the shear diagram has the same scale as the force polygon, its scale is 1″ = 8,000#. Fig. 22 as shown here has, of course, been reduced in size from the original drawing.

Now let us compute the value of the bending moments by mathematics.* The value of the bending moment at any section of a beam is equal to the algebraic sum of the moments of the forces on either side of the section. For convenience we may say, *the value of the bending moment is equal to the moments of the reactions minus the moments of the loads to the left of the section.* The value of the bending moment is easily found, for all that is necessary is to select some particular section of the beam and write the value in accordance with the above rule. The value of the bending moment is zero at R_1 as there are no forces to the left. The values at other sections are as follows:

Bending moment at load $AB = 7{,}200 \times 6 = 43{,}200'\#$

Bending moment at load $BC = (7{,}200 \times 12) - (4{,}000 \times 6) = 62{,}400'\#$

Bending moment at load $CD = (7{,}200 \times 16) - [(4{,}000 \times 10) + (10{,}000 \times 4)] = 35{,}200'\#$

Bending moment at $R_2 = (7{,}200 \times 20) - [(4{,}000 \times 14) + (10{,}000 \times 8) + (2{,}000 \text{ x } 4)] = 0$

The shear diagram shows that the shear passes through zero, that is, from a plus quantity to a minus quantity, under the load BC, and this is the section at which we would expect the bending moment to be a maximum. Note that this is shown to be true in the moment diagram, its value being 62,400′#.

36. Overhanging Beam with Concentrated Loads. Fig. 23 (*a*) represents a beam overhanging the right-hand support. There are three concentrated loads. The shear and moment diagrams are drawn in the same manner that has been described previously. The force polygon is shown at Fig. 23 (*b*) and the funicular polygon at Fig. 23 (*d*). The closing ray *oe* determines the values of R_1 and R_2, and their magnitudes may be found by scaling the force polygon.

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* See Arts. 29 and 30, *Simplified Engineering for Architects and Builders.*

By mathematics,

$$20\,R_2 = (10{,}000 \times 8) + (6{,}000 \times 14) + (8{,}000 \times 24)$$
$$20\,R_2 = 356{,}000 \quad \text{and} \quad R_2 = 17{,}800\#$$
$$20\,R_1 + (8{,}000 \times 4) = (10{,}000 \times 12) + (6{,}000 \times 6)$$
$$20\,R_1 = 124{,}000 \quad \text{and} \quad R_1 = 6{,}200\#$$

Check
$$10{,}000 + 6{,}000 + 8{,}000 = 17{,}800 + 6{,}200$$
$$24{,}000 = 24{,}000$$

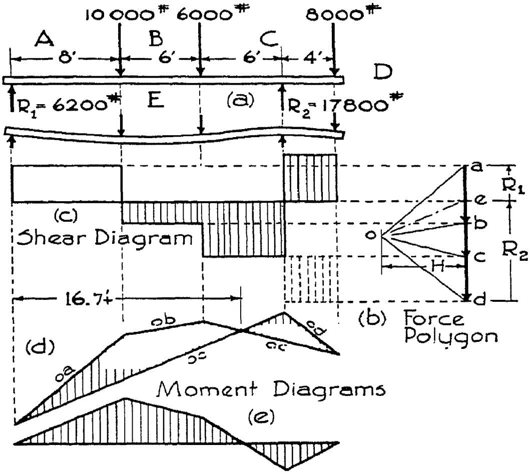

FIG. 23

The shear diagram is drawn as has been described, with a slight modification. Following the conventional method of representing the values of the shear, the upward or positive values are shown above the base line and the downward or negative values are below. For instance, the value of the shear between *EA* and *AB* is *ea*, an upward or positive value. See Fig. 23 (*b*). Between the forces *AB* and *BC*, and also between *BC* and *DE*, the values of the shear are downward or negative, and are hence shown below the base line. The next section is between *DE* and *CD* and the value is $(DE + EA) - (AB + BC)$, or *DC*. Now *dc* in the force polygon reads *upward*, a positive value, and, consequently, this value will be shown *above* the base line. As has been seen, the values of the shear at the various sections in the beam are merely projected horizontally from the force polygon. Therefore, the *magnitude* of *dc* is found from the force polygon but it is placed *above* the base line in accordance with the conventional method. See Fig. 23 (*c*).

On inspecting the shear diagram it is observed that the shear passes through zero at two points, namely, under the force *AB*, and at *DE*, the right-hand reaction. This indicates that the bending moment will approach maximum values at each of these sections. To determine which is the greater numerically we can scale the moment diagram or compute the values mathematically. Actually, the moment under *AB* is a positive moment and the moment at the reaction is negative. A positive bending moment is considered as one which produces compressive stresses in the upper fibers of the beam and tension in the lower fibers. Sometimes we say the beam is concave upward if the bending moment is positive. The exact opposite is true for negative bending moments. Beneath the beam is an exaggerated diagram showing the curve the beam tends to assume. The section at which the beam changes from a positive to a negative bending moment is called the *inflection point*. At this section the value of the bending moment is zero. Its position may be found by scaling the distance in the bending-moment diagram, or it may be computed by mathematics.

Using the methods described in Art. 35, the values of the shear and bending moment at various sections of the beam are computed mathematically as follows. The subscripts "$x = 1$," "$x = 9$," etc., indicate the distances from the left reaction at which the shear or moment is computed.

Shear.

$$\text{Shear}_{(x=1)} = 6{,}200 - 0 = +6{,}200\#$$
$$\text{Shear}_{(x=9)} = 6{,}200 - 10{,}000 = -3{,}800\#$$
$$\text{Shear}_{(x=15)} = 6{,}200 - (10{,}000 + 6{,}000) = -9{,}800\#$$
$$\text{Shear}_{(x=21)} = (6{,}200 + 17{,}800) - (10{,}000 + 6{,}000) = +8{,}000\#$$

Bending Moment.

$$\text{Bending Moment}_{(x=8)} = 6{,}200 \times 8 = +49{,}600'\#$$
$$\text{Bending Moment}_{(x=14)} = (6{,}200 \times 14) - (10{,}000 \times 6) = +26{,}800'\#$$
$$\text{Bending Moment}_{(x=20)} = (6{,}200 \times 20) - [(10{,}000 \times 12) + (6{,}000 \times 6)] = -32{,}000'\#$$

The position of the inflection point can be computed by calling the unknown distance from the left reaction x. Since the value of the bending moment at this section is zero, the value of the moment is written, equated to zero, and the value of x computed thus:

$$(6{,}200 \times x) - \{[10{,}000 \times (x - 8)] + [6{,}000 \times (x - 14)]\} = 0$$

or

$$9{,}800x = 164{,}000$$

and

$$x = 16.7+'$$

One of the important reasons for determining the position of the inflection point is in connection with reinforced-concrete beams, for at this point the reinforcing rods used to resist tensile stresses are turned up.

Although the bending-moment diagram shown in Fig. 23 (*d*) determines the values of the moments at any section of the beam, the conventional method of representation is shown at Fig. 23 (*e*). It is readily constructed by drawing a base line and plotting the values of the moments above and below, depending, of course, upon the character of the value of the moment, positive above and negative below the base line. The magnitudes of these values may be scaled directly from the vertical distances in Fig. 23 (*d*).

37. **Simple Beam with Uniformly Distributed Load.** Exact diagrams for shear and moments for beams having uniformly distributed loads may not be drawn by the graphical methods previously described. It is possible, however, to assume the uniformly distributed load to be made up of a number of equal concentrated loads, and approximate diagrams are thus obtained. The algebraic or analytical solution for such problems is invariably more direct and, as compared with the graphical methods, requires less time in solution.*

Fig. 24 (*a*) represents a simple beam 18′0″ in length having a uniformly distributed load W of 18,000#. To draw the shear and moment diagrams by graphical methods assume the load to be composed of a number of smaller concentrated loads, in this instance 9 loads of 2,000# each. Following the methods used in the previous problems, the force polygon, moment and shear diagrams are constructed as shown. The shear and moment diagrams, however, are not the true diagrams for the uniformly distributed load. Note that the diagram for shear consists of a number of "steps." If a greater number of concentrated loads had been taken, a greater number of steps would have resulted. The true diagram would consist of no steps whatsoever but would be a straight line as indicated.

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* See Art. 36, *Simplified Engineering for Architects and Builders.*

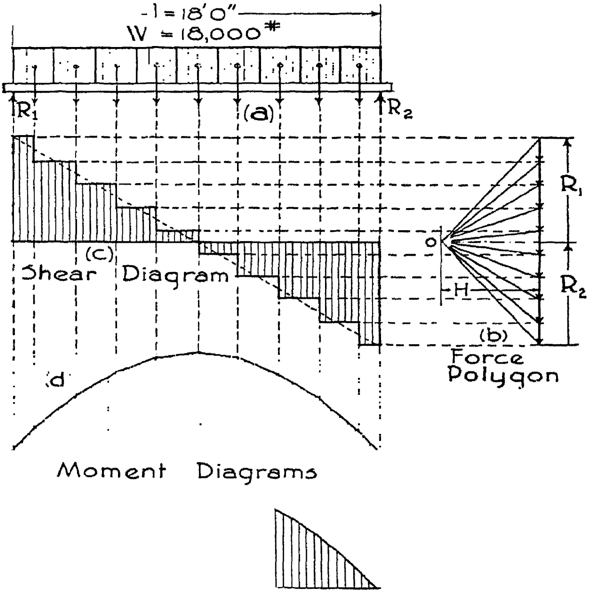


FIG. 24

Fig. 24 (*d*) is the moment diagram drawn as a result of assuming a number of concentrated loads. The true curve for a uniformly distributed load is a parabola as shown at Fig. 24 (*e*).

By mathematics, the total load $W = 18{,}000\#$, and, since the reactions are equal, $R_1 = R_2 = 18{,}000 \div 2 = 9{,}000\#$. Hence the maximum shear is 9,000# and occurs at each reaction. The shear passes through zero at the center of the span, at which point the bending moment will have a maximum value:

$$M_{(x=9)} = (9{,}000 \times 9) - (1{,}000 \times 9 \times 4.5) = 40{,}500'\#$$

Since this loading occurs so frequently, it is convenient to remember the value of the maximum bending moment for this beam, namely, $M = \frac{Wl}{8}$, or

$$M = \frac{18{,}000 \times 18}{8} = 40{,}500'\#$$

38. Overhanging Beam with Uniformly Distributed Load. A beam overhanging both supports, having a uniformly distributed load, is shown in Fig. 25 (*a*). The method of assuming a number of concentrated loads in place of the uniformly distributed load is again employed and the shear and moment diagrams constructed as has been explained. Whereas the shear and moment diagrams thus constructed are true diagrams for the assumed concentrated loads, they are only approximate for the actual distributed load. For the beam shown, note that the maximum shear occurs at R_1. The shear diagram shows that the shear passes through zero at three points and at these points the moment will have maximum

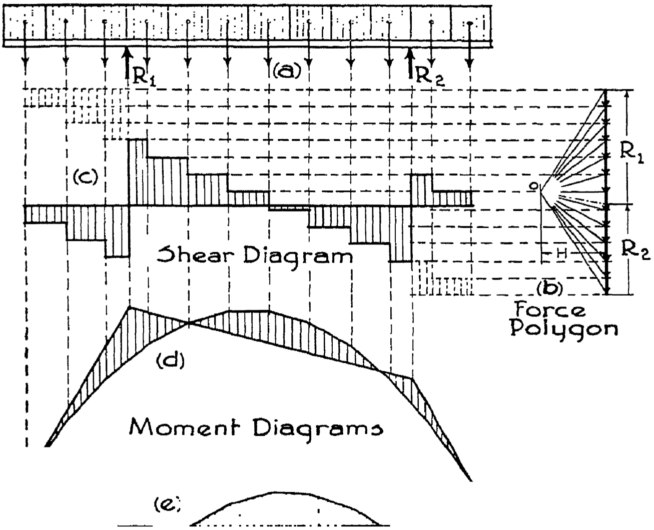


FIG. 25

values, one of which is positive, the other two being negative. Note also that the moment diagram passes through zero at two points and consequently there are two inflection points.

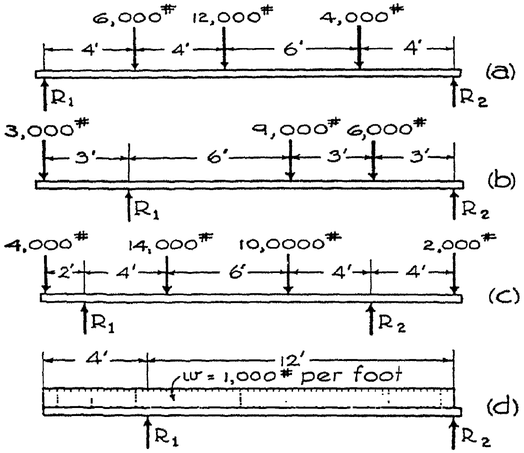


FIG. 26

To review the principles previously explained, give values to the length of the beam and overhangs, assume a uniformly distributed load per linear foot and compute the various values of the shear and moment.*

Problems 38-A-B-C-D. For the beams shown in Figs. 26 (*a*), (*b*), (*c*) and (*d*), draw the shear and moment diagrams by graphical methods and check the results by mathematics.

* See Art. 32, *Simplified Engineering for Architects and Builders.*

PART II

STRESSES IN ROOF TRUSSES

CHAPTER 5

TYPES OF ROOF TRUSSES

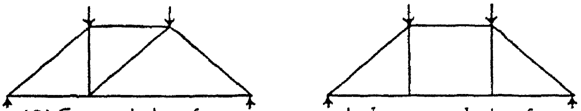
39. **Framed Structural Members.** When it is desired to construct a roof over a building which has no interior supports, and whose exterior walls or columns are more than about 40′ apart, it is often more economical to resort to some system of framing other than simple beams. The frame used for this purpose is called a roof truss. A *truss* is a framed structure, usually supported at its ends only, having a system of straight members so arranged and secured to each other that the stresses transmitted from one member to another are longitudinal only, that is, tension or compression. A triangle is the only polygon whose shape is incapable of being changed without changing the length of one or more of its sides, consequently a truss is composed, fundamentally, of a system of triangles.

Although the common practice of connecting angles by means of gusset plates and rivets, in steel trusses, forms a rigid connection, a truss does not depend upon the rigidity of its joints for stability. Theoretically, the members are assumed to be joined together at the joints by frictionless pins, although in practice this condition never exists. Since, whenever possible, the members are so arranged that the axes of all members at each joint meet in a common point, none of the stresses in the members can cause a moment about this point. Therefore, the stresses are axial, compression or tension, and are equal throughout the length of the members.

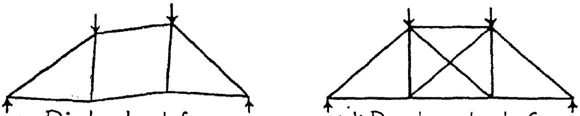
40. **Types of Frames.** A *complete frame* is one which is composed of the least number of members required to form a structure made up entirely of triangles, Fig. 27 (*a*).

An *incomplete frame* is a frame not composed entirely of triangles, Fig. 27 (*b*). For symmetrical loading this truss is stable, but if the loading is unsymmetrical, distortion will occur and failure may result, Fig. 27 (*c*). An incomplete frame is considered to be unstable and is always to be avoided.

A *redundant frame* is a frame containing a greater number of members than are required to form a frame of the minimum number of triangles, Fig. 27 (*d*). In the truss illustrated two diagonals are shown in the central portion; one of the diagonals is called a redundant member. In practice, however, these two diagonals, composed of rods, are often used. Since a rod is capable of resisting only tensile forces, only one member will act at one time. For unsymmetrical loading the member resisting a tensile force will be stressed while the other diagonal member will have no stress. If only one diagonal is employed, it must be capable of resisting either compression or tension, depending on the relative magnitudes of the applied loads.



(a) Complete frame (b) Incomplete frame



(c) Distorted frame (d) Redundant frame

FIG. 27

41. **Parts of a Truss.** In general, a truss is composed of the upper and lower chords and web members, Fig. 28. The *upper chord* consists of the uppermost line of members extending from one end of the truss to the other through the apex. For triangular trusses the greatest stress in the upper chord usually occurs in the member adjacent to the support. The *lower chord* of a truss is composed of the lower line of members and extends from one end of the truss to the other. As in the upper chord, the greatest stress in the lower chord of triangular trusses is usually found in the member adjacent to the support.

The members connecting the joints of the upper and lower chords are the *web members*, and, depending on their positions, are called *verticals* and *diagonals*. On a basis of character of stress, compression members of a truss are called *struts* and those members having tensile stresses are *ties*. The joint at the support of a triangular truss is called the *heel joint*, and the joint at the ridge is the *peak*. A *panel point* is the name given to the joints at which web members are connected to the chords.

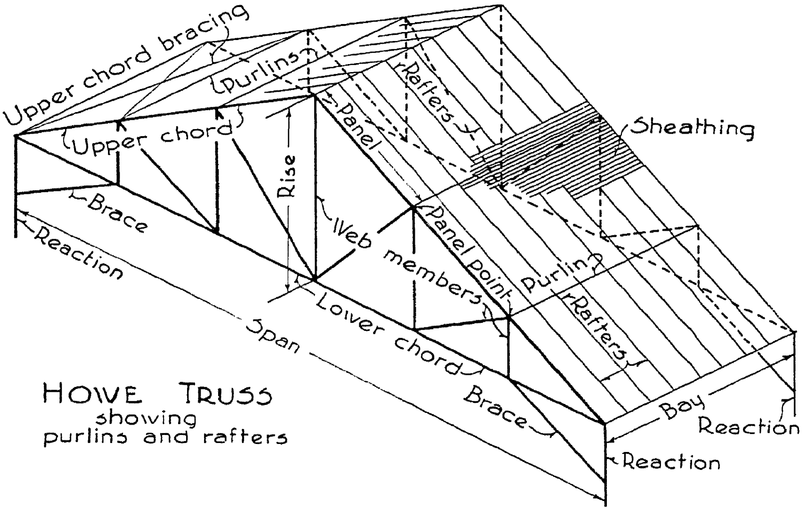


FIG. 28

A method commonly used to designate the various members of a truss is shown in Fig. 29. The letters U and L are placed at the joints of the upper and lower chords with numerical subscripts showing the number of panels from the left support. Thus, L_0U_1 and U_1U_2 are upper chord members and L_0L_1, L_1L_2 are lower chord members. The verticals are U_1L_1, U_2L_2, etc., and U_2L_1 and U_3L_2 are diagonals.

42. Pitch of Trusses. The *span* of a truss is the distance between the end joints. When a truss is supported by walls, the span is usually considered to be the distance from center to center of bearings. If the truss frames into columns at its ends, the span is generally taken as the distance between the column faces. The vertical distance from the apex to a line joining the supports of a truss is called the *rise.*

The *pitch* of a roof may be expressed in different ways. A common method is to give the ratio of rise to span. For instance, a half-pitch indicates a truss having a rise of one-half the span, or a span of 48′0″ and a rise of 12′0″ is known as a quarter-pitch. Another method of designating the pitch is to give the angle the upper chord makes with the lower chord, as for instance a 30° or 45° pitch. For symmetrical trusses the *run* is one-half the span, and frequently the pitch is expressed as a ratio of the rise to the run, usually in inches. Frequently this ratio is called the *slope.*

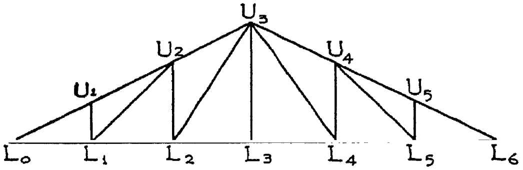


FIG. 29

Consider a truss having a span of 40′0″ and a rise of 20′0″. In accordance with the methods of designation previously given, the pitch would be expressed as (1) half-pitch, (2) 45° pitch and (3) 12 in 12 slope. Table I gives various roof pitches and their equivalents in degrees and slopes.

Table I. Pitch of Roofs

Pitch....	1/8	1/6	1/5	1/4	1/3.46	1/3	1/2
Degrees..	14° 3′	18° 26′	21° 48′	26° 34′	30° 0′	33° 40′	45° 0′
Slope....	3 in 12	4 in 12	4.8 in 12	6 in 12	6.92 in 12	8 in 12	12 in 12

Roofs are known as *flat* or *pitched.* When the rise is not greater than about 2″ per horizontal foot, the roof is said to be flat. Pitched roofs include all roofs whose pitch exceeds this ratio.

Very often the pitch of a roof is determined by aesthetic considerations, or it may be that the pitch is limited by the type of roofing material. For commercial buildings the pitch is usually determined by economic considerations. It is generally considered that a rise of about 6″ to the foot is probably the most economical for average spans. Roofs with steep pitches must resist greater wind loads and require the use of more roofing material, while pitches less steep result in increased stresses in the truss members. It should be remembered that relatively short compression members effect a saving of material, whereas the length of a member in tension is not an element in determining its cross section.

43. **Panels.** A *panel* is that portion of a truss which lies between two adjacent joints of the upper chord. The beam spanning from truss to truss framing into the upper chord is called a *purlin*. The portion of a roof included between trusses is known as a *bay*, Fig. 28. Since the purlins extend from truss to truss, the width of a bay corresponds to the length of a purlin.

Regardless of the framing employed, the roof load is transferred to the truss at the panel points, usually by the purlins. One of the most common types of roof framing is indicated in Fig. 28 and Fig. 30 (*b*). In this instance the roof load is transferred from the sheathing to the rafters, from the rafters to the purlins and from the purlins to the trusses at the panel points. Another method, shown in Fig. 30 (*a*), consists in running the sheathing from purlin to purlin, the rafters being omitted. For this type of framing the saving effected by the omission of rafters is offset by the required thickness of the plank sheathing. Still another framing plan, indicated in Fig. 30 (*c*), consists in running the rafters from truss to truss, omitting the purlins. Here the loads are transferred from sheathing to rafters and from the rafters to the upper chord of the truss. This results in the upper chord being subjected to bending stresses as well as direct compression due to the forces transferred from other members of the truss.

From the above discussion it is seen that the type of truss to be used for a building depends first of all upon the number of panels. The number of panels in the upper chord is determined by the allowable span of the materials supporting the roofing and the length of the upper chord between panel points. It is advisable to have equal panels and also to have them symmetrical with respect to the center line of the truss. For the three methods indicated in Fig. 30 (*a*), (*b*) and (*c*), the panel lengths generally used are respectively 5′0″ to 8′0″, 8′0″ to 11′0″ and 5′0″ to 6′0″. Fig. 30

represents types of timber framing, but for steel construction similar framing methods are employed. Obviously, the length of

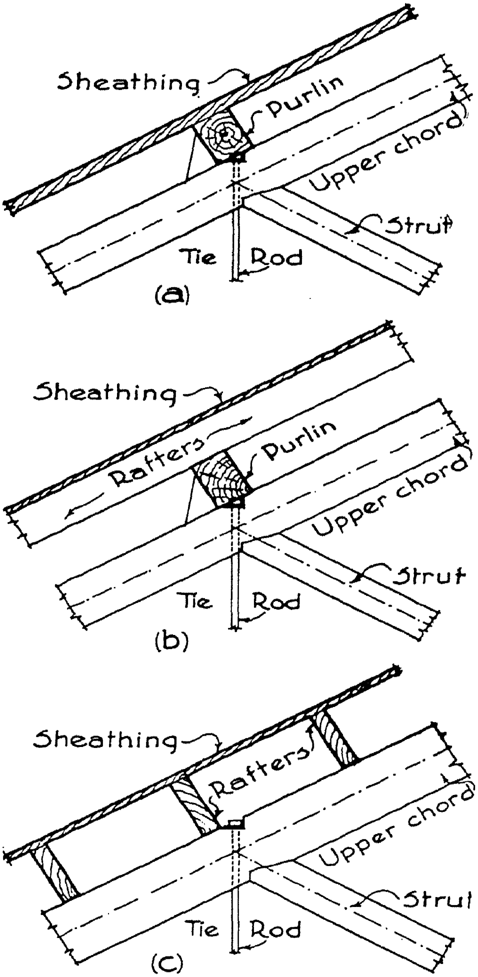


Fig. 30

panels depends somewhat upon whether the construction is of timber or steel. When roof loads are transferred to steel trusses by purlins at panel points only, the panel lengths of the upper chord are generally about 8′.

44. **Spacing of Trusses.** For many buildings the arrangement of the plan, window spacing, piers and columns, determine the location of the trusses. When the spacing of trusses is optional with the designer, the length of the building may determine the number of trusses. Although a relatively close spacing results in smaller loads per truss and consequently smaller sections for purlins and truss members, a greater number of trusses increases the expense involved in fabrication. For the trusses generally used, the spacing is between 14′0″ and 22′0″. Trusses having spans up to 30′0″ are spaced approximately 12′0″ on centers, and for greater spans, up to 60′0″, the spacing is about 17′0″. The maximum spacing for commonly employed trusses is about 23′0″.

45. **Types of Trusses.** Several factors are to be considered in selecting the particular type of truss to be used. First of all is the contour or shape of the roof. This is generally determined by architectural requirements. The material of which the truss is to be made is also a governing factor. Trusses made entirely of steel are in common use today, whereas in the past timber trusses using wrought-iron rods for certain tensile members were used extensively. Owing to the difficulty in designing connections, many trusses are practical only when constructed of steel. A truss which is to be exposed, as frequently occurs in ecclesiastical architecture, is invariably constructed of timber, although sometimes the structural elements are of steel enclosed with timber. The span of the truss determines the number of panels, and the number of divisions of the upper chord determines, to some extent, the type of truss to use. Some types of trusses are more economical than others and, in this respect, experience and practical considerations constitute the best guide.

It is always advisable to avoid the use of long members. This is particularly true with respect to compression members. In designing a strut its length is a governing factor and by keeping it as short as possible the tendency to bend is reduced. A long member in tension has a tendency to sag due to its weight.

The use of a horizontal lower chord is to be preferred. If added ceiling height is desirable, a truss may be "cambered," but this adds appreciably to the stresses developed within the members.

Likewise, a truss with less pitch has increased stresses as compared with a similar truss of equal span and a greater pitch.

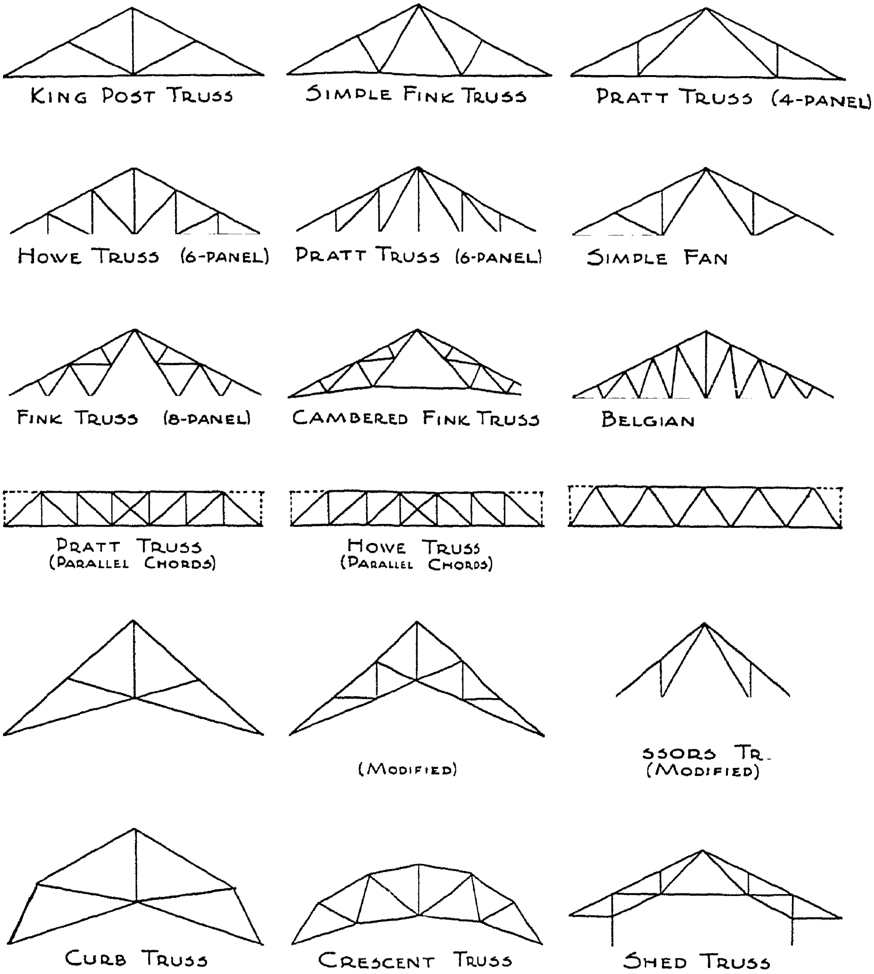


Fig. 31

Some of the common types of roof trusses are illustrated in Fig. 31.

CHAPTER 6

LOADS ON ROOF TRUSSES

46. **Dead Loads.** To determine the stresses in truss members, the first step is to compute the loads the truss will be required to support. In general they consist of *dead loads* and *live loads*. The former includes the weight of all construction materials supported by the truss, and the latter includes loads resulting from snow and wind.

The following items constitute the materials to be considered in computing the dead loads: roof covering; purlins, rafters and sway bracing; ceiling and any suspended loads; and the weight of the truss. Obviously, all these loads cannot be exactly determined before the truss is designed, but all may be checked later to see if a sufficient allowance has been made. The dead loads are downward vertical forces, and hence the reactions or forces supporting the truss are also vertical with respect to these loads.

47. **Roofing Materials.** The materials constituting the roof covering, such as slate and sheathing, are first determined. Table II gives approximate weights, in pounds per square foot, of some of the materials commonly used. When a type of construction not listed is to be employed, the designer should communicate with the manufacturer to determine the weight allowance to be made for his material.

48. **Rafters and Purlins.** The weight allowance for wood roof rafters is given in Table III. Depending on conditions, wood and steel purlins average from 1.5 to 4.5#/ft^2 of roof surface. Since both the rafters and purlins are beams, their sizes are readily computed and therefore their exact weight may be determined before beginning the design of the truss. It is convenient to estimate their weight in *pounds per square foot* of roof surface so that it may be added to other dead loads in determining the downward forces at the panel points.

When no ceiling construction exists, it is generally advisable to

Table II. Approximate Weights of Roofing Materials

Roofing material	Weight per square foot
	lb.
Shingles	
Wood	3.0
Asbestos	5.0–6.0
Asphalt, slate-covered	2.0
Slate	
3/16″ thick	7.0
1/4″ thick	10.0
3/8″ thick	12.0–14.0
Tile, clay	
Flat	12.0–16.0
Spanish	10.0–14.0
Built-up roofing	
4-ply felt	4.0–5.0
5-ply felt	6.0–8.0
Corrugated iron	
20-gage	2.0
18-gage	3.0
Tin plate	1.0
Copper, sheets	1.0
Lead, sheets	7.0
Sheathing, wood	
Spruce and hemlock, 1″ thick	3.0
Yellow pine, 1″ thick	4.0
Nailing concrete, per inch of thickness	8.0
Concrete slab, cinder, per inch of thickness	9.0
Gypsum slab, per inch of thickness	8.0
Plaster ceiling	10.0

Table III. Weights of Roof Rafters, per Square Foot of Roof Surface*

Kind of timber	White pine, spruce and hemlock				Yellow pine			
Spacing	12″	16″	20″	24″	12″	16″	20″	24″
in	lb	lb	lb	lb	lb	lb	lb	lb
2 x 4	1.3	0.9	0.8	0.6	1.7	1.2	1.0	0.8
2 x 6	1.9	1.4	1.1	1.0	2.5	1.9	1.5	1.3
2 x 8	2.5	1.9	1.5	1.3	3.4	2.5	2.0	1.7
2 x 10	3.2	2.4	2.0	1.7	4.3	3.2	2.6	2.2
2 x 12	3.9	2.9	2.3	2.0	5.2	3.9	3.1	2.6

* Taken from Kidder-Parker, *Architects' and Builders' Handbook*, John Wiley & Sons, New York.

install bracing at the lower chord of the truss to maintain the trusses in vertical planes. Rigid roof construction may serve as sufficient bracing for the upper chord. However, for trusses having spans of 60′ and over, bracing is invariably used. The weight of bracing seldom exceeds 1#/ft^2 of roof surface.

49. **Suspended Loads.** When a ceiling is suspended from the lower chord, its weight must be taken into consideration in the design of the truss. An allowance of 10#/ft^2 is generally assumed as the weight of its construction. Other suspended loads consist of balconies, large lighting fixtures, mechanical equipment, etc. Their weights should be accurately computed and their exact positions determined.

Table IV. Weights of Timber Trusses per Square Foot of Roof Surface*

Span	½ Pitch	⅓ Pitch	¼ Pitch	Flat
ft	lb	lb	lb	lb
Up to 36	3	3½	3¾	4
36 to 50	3¼	3¾	4	4½
50 to 60	3½	4	4½	4¾
60 to 70	3¾	4½	4¾	5¼
70 to 80	4¼	5	5½	6

* For cambered trusses increase 30%.

50. **Weights of Trusses.** The true weight of a truss cannot be exactly determined until it is completely designed. Several formulas have been prepared by means of which the approximate weight may be computed. Such formulas include the span, spacing, rise and in some instances the total capacity of the truss. However, no two formulas give the same result since they are all empirical. Tables IV and V, for timber and steel trusses respectively, give their approximate weights *per square foot of roof surface.* These tables have been compiled from a comparison of formulas and from the weights of actual trusses. When the design of the truss is completed the actual weight should be compared with the weight allowance used in design. For the usual conditions met with in practice, it will seldom be necessary to redesign the truss.

It is customary to add the weight due to the truss to the panel loads on the upper chord. A more exact method would be to appor-

Table V. Weights of Steel Trusses per Square Foot of Roof Surface*

Span	½ Pitch	⅓ Pitch	¼ Pitch	Flat
ft	lb	lb	lb	lb
Up to 40	5.25	6.3	6.8	7.6
40 to 50	5.75	6.6	7.2	8.0
50 to 60	6.75	8.0	8.6	9.6
60 to 70	7.25	8.5	9.2	10.2
70 to 80	7.75	9.0	9.7	10.8
80 to 100	8.5	10.0	10.8	12.0
100 to 120	9.5	11.0	12.0	13.2

* For cambered trusses increase 30%.

tion a part of such loads to the panel points of the lower chord. This, however, is customary only in trusses having exceptionally long spans.

51. **Snow Loads.** The weight allowance for snow loads depends primarily upon the locality in which the building is to be erected. Freshly fallen dry snow weighs from 6 to 8#/ft^3, and hail and sleet may weigh as much as 30#/ft^3. When there is no building code to determine the snow load allowance, Table VI may be used with safety. Note that this table gives the weight *per square foot of roof surface,* whereas some building codes state the snow load in pounds per square foot *of horizontal projection.*

Table VI. Snow Loads for Roof Trusses* in Pounds per Square Foot of Roof Surface

Locality	Slope of roof				
	45°	30°	25°	20°	Flat
Northwestern and New England States	10–15	15–20	25–30	35	40
Western and Central States	5–10	10–15	20–25	25–30	35
Southern and Pacific States	0–5	5–10	5–10	5–10	10

* Taken from Kidder-Parker, *Architects' and Builders' Handbook*, published by John Wiley & Sons, New York.

Since the snow load acts vertically, some designers add this load to the dead load for the purpose of drawing a vertical load stress diagram. Frequently, however, it is desirable to know the stresses in members due to snow loads alone. In this case a separate snow load stress diagram may be drawn, or the stresses may be computed from stresses due to dead loads as they are directly proportional.

52. **Wind Loads.** Although the direction of the wind is a variable, for the purpose of computations it is assumed to act horizontally. It is generally accepted that a pressure of 30#/ft^2 perpendicular to the pitch of the roof is seldom exceeded. When the wind strikes an inclined surface it is deflected, and in computing its force exerted on a roof the normal component, that is, the component perpendicular to the roof surface, is the force to be considered. Fig. 32 (*a*) represents a small truss with the resultant horizontal wind pressure. Fig. 32 (*b*) shows this horizontal force with two components, one normal and the other parallel to the roof surface. From this diagram it is seen that the normal wind pressure varies with the pitch. Table VII, based on a maximum pressure of 30#/ft^2, gives normal pressures for various pitches. The

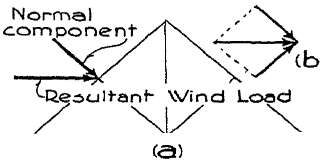


FIG. 32

Table VII. Wind Pressure on Roof Surfaces

Pitch of roof	Normal pressure
Degrees	Pounds per square foot
10	10
15	15
20	18
25	22
30	24
35	26
40	27
45	28
50	29
55	29
60	30
For pitches over 60° use 30#	

values in this table are established in accordance with an accepted formula and not by a simple parallelogram of forces. Experiment has verified their accuracy, and they may be used with safety. In determining the stresses in truss members due to wind forces, the forces are assumed to act on only one side of the truss at one time and the direction of the wind load is taken as perpendicular to the roof surface.

53. **Computation of Truss Loads.** After the type of truss and the roof construction have been determined, the next step is to compute the loads that the truss will be required to support. This is an important step, and although the values cannot be determined exactly they must be established within reasonably accurate limits. The usual method of procedure is to find the number of square feet of roof surface tributary to a panel point and to multiply this number by the load per square foot. These loads are dead load, snow load and wind load. Since the dead load and snow load act vertically they are often combined. Another method that may be used is to consider simultaneously the dead load and a vertical equivalent snow and wind load. In this case, only one stress diagram is required. Although not so accurate as the former method, the results obtained are adequate for the most common trusses. Equivalent vertical wind and snow loads are given in Art. 54. To illustrate the computations necessary in determining panel loads, the following example is given.

Let it be required to compute the dead, snow, wind and ceiling loads for the timber truss indicated in Fig. 33 (*a*).

DATA: A timber truss having a span of 41′6″ and a pitch of 30° is to be built in one of the Central States. The roof surface is of ⅜″ slate laid on 1″ sheathing. The rafters are of hemlock, 2″ x 6″ – 16″ o.c. and the purlins are 8″ x 12″. A plaster ceiling is suspended from the lower chord. The spacing of the trusses is 16′0″.

A span of 41′6″ and a pitch of 30° gives an upper chord of 24′0″ in length and, as the Howe truss illustrated has the upper chord divided into thirds, the panel points are 8′0″ apart. Since the trusses are spaced 16′0″ on centers, the roof area transferring a load to one panel point is, therefore, 8′ × 16′ or 128ft². This area is indicated by the hatched section in Fig. 33 (*b*).

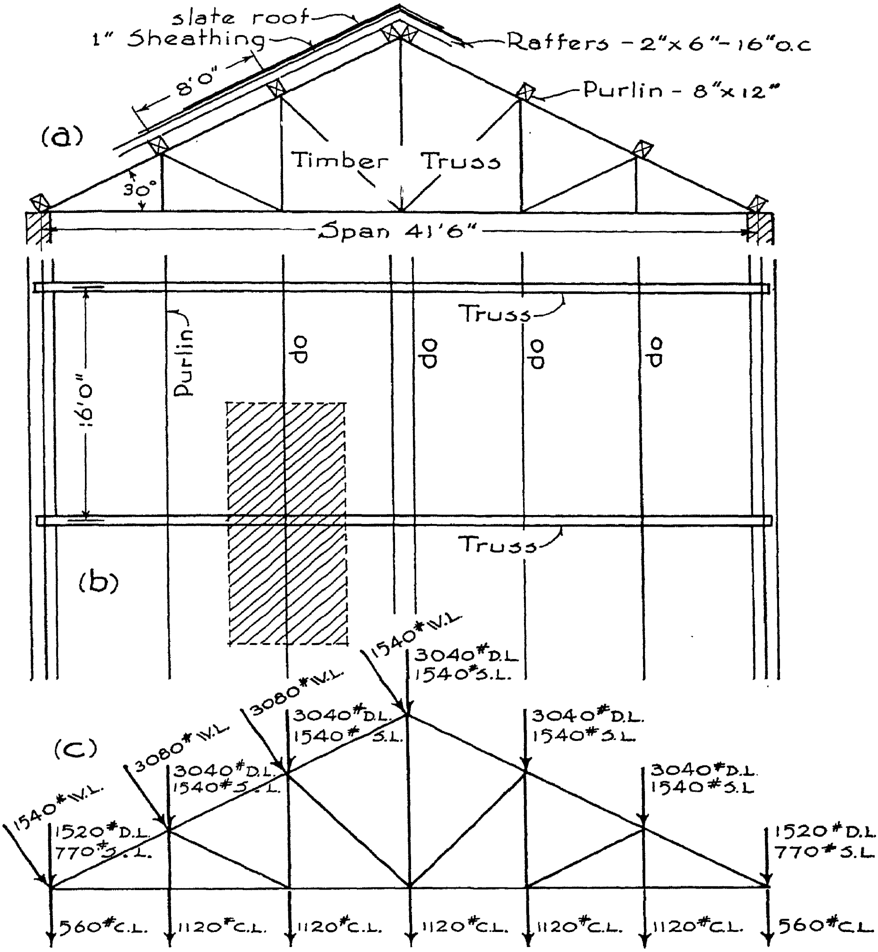


FIG. 33

The weight of one 8" x 12" purlin, 16'0" in length, assuming the weight of timber to be 40#/ft³ is computed thus:

$$\frac{8 \times 12}{144} \times 16 \times 40 = 427\#$$

The weight of one purlin divided by the number of square feet at one panel point gives the equivalent weight per square foot of roof surface due to the purlin, or 427 ÷ 128 = 3.3#

Tabulating the dead loads,

⅜″ slate roofing	= 12.0#	Table II
1″ sheathing	= 3.0	Table II
2″ x 6″ rafters, 16″ o.c.	= .4	Table III
8″ x 12″ purlin	= 3.3	(See above)
Truss	= 4.0	Table IV
Total	= 23.7#/ft^2	

Total dead panel load = 128 × 23.7 = 3,033.6, say 3,040#
Snow panel load = 128 × 12 = 1,536, say 1,540#, Table VI
Wind panel load = 128 × 24 = 3,072, say 3,080#, Table VII

The area per panel point on the lower chord is approximately 7 × 16 = 112ft^2, hence the panel load for the suspended ceiling is 112 × 10 = 1,120#, Table II.

The above simple computations show the method of computing panel loads. Fig. 33 (*c*) shows these loads diagrammatically. Note that the loads on the joints at the supports receive only half-panel loads due to the framing. It will be found later that the loads at the truss supports have no effect on the stresses developed in the truss members. They are shown in Fig. 33 (*c*) since *all* the truss loads are given for the purpose of illustration. Also, it is important to know the total load on a truss in designing truss bearings or column connections.

There are six panels in the upper chord, hence the total dead load on the truss is 6 × 3,040# or 18,240#; the total snow load is 6 × 1,540# or 9,240#; the total ceiling load is 6 × 1,120# or 6,720#, and 3 × 3,080# = 9,240#, the total wind load. Since the dead, snow and ceiling loads are vertical, and as the truss is symmetrically loaded, they may be added together to find the reactions due to the vertical loads, the reactions, of course, being equal. On the other hand, the wind load acts at an angle with the vertical, and consequently the reactions due to the action of the wind are not vertical. Wind load reactions are discussed in Chapter 7. Note that the loads shown in Fig. 33 (*c*) include all the *loads* on the truss, but the reactions, or forces supporting the truss, are purposely omitted to avoid confusion.

54. **Equivalent Vertical Loads.** It is improbable that a maximum snow load and maximum wind load will occur simultaneously to exert pressure on a truss. The combinations of loads most likely to occur, and as commonly used by designers, are as follows:

1. Dead load and maximum snow
2. Dead load, maximum wind and minimum snow
3. Dead load, minimum wind and maximum snow

To determine accurately the greatest stresses that the various truss members will be required to resist, it is necessary to construct separate stress diagrams for dead, snow and wind loads and to tabulate the stresses in each member due to these loads, using for the design load the combination giving the greatest stress. It is customary to consider one-half the maximum snow and wind loads as the minimum values. This method of determining the maximum stress in truss members is illustrated by the example given in Art. 94.

Instead of the above method, a procedure greatly simplifying the work is to use a vertical load in which are combined the dead load and a vertical equivalent snow and wind load. This method may be used in safety for trusses whose spans are not excessive. Since in using this method all the loads are considered as being vertical, only one stress diagram need be drawn and the work is consequently simplified. When a stress diagram for wind loads alone is drawn, the wind is assumed to exert pressure on only one side of the roof. However, in using the values of equivalent vertical loads for wind and snow combined, the forces are assumed to be

Table VIII. Equivalent Vertical Loads for Combined Snow and Wind Loads in Pounds per Square Foot of Roof Surface

Locality	Slope of roof				
	60°	45°	30°	20°	Flat
Northwestern and New England States	28	26	24	35	40
Western and Central States	28	26	24	30	35
Southern and Pacific States	28	26	24	30	30

uniformly distributed over the entire roof surface. Table VIII gives minimum values for combined snow and wind loads. The simplicity of this method is illustrated in the example shown in Art. 95 as well as in the design of the timber and steel trusses given in Arts. 107 and 125.

PROBLEM 54-A. An eight-panel steel Fink truss having a span of 59′0″ is to be built in one of the New England States. See Fig. 31. Compute the dead, snow, wind and suspended loads in accordance with the following data:

Pitch of roof	30°
Roofing surface	¼″ slate
Sheathing	3″ planking
Purlins	8″ x 12″
Suspended plaster ceiling	
Trusses spaced 17′0″ o.c.	

PROBLEM 54-B. Compute the panel loads on the upper chord of the truss given in Problem 54-A, using equivalent vertical loads for snow and wind combined.

CHAPTER 7

REACTIONS

55. **Reactions—Supporting Forces.** A truss is a framed structure subjected to *external forces*, the stresses in the truss members being dependent on these forces. The external forces are the loads, dead, snow and wind, and the reactions or forces that support the truss. For vertical loads the reactions are vertical. For wind loads the directions of the reactions are not vertical but are determined by the pitch of the roof and the construction of the connections at the supports. In investigating the stresses in a truss, it is essential that the reactions be determined accurately. To find the stresses in truss members by graphical methods, we construct what is known as a stress diagram. *To make the stress diagram the very first step is to draw a force polygon of the external forces.* Since these forces consist of the loads and reactions, and as the methods of computing the loads have previously been discussed, let us now investigate the reactions. As for their location, we know only a point in their lines of action, that is, the point of support. Their magnitudes and directions are to be found.

56. **Reactions for Vertical Loads, Trusses Loaded Symmetrically.** Since for vertical loads the reactions are vertical, determining the reactions is a very simple matter. The system of forces is nothing more than a number of parallel forces in equilibrium. See Arts. 26 and 27. If the truss has vertical loads and is symmetrically loaded, which is a very common condition, the reactions are equal and vertical and each reaction is equal in magnitude to one-half the sum of the loads.

Fig. 34 (*a*) represents the outline of a truss, the web members being omitted to avoid confusion. This truss has six equal panels, the panel loads being 2,000# each with the exception of the loads over the supports which, of course, are half-panel loads or 1,000# each. The total vertical load therefore is

$$(5 \times 2{,}000) \qquad (2 \times 1{,}000) = 12{,}000\#$$

As the truss is loaded symmetrically each reaction is equal to 12,000 ÷ 2, or 6,000#, their directions being vertical.

In this problem the forces consist of the downward forces, *BC*, *CD*, *DE*, *EF*, *FG*, *GH*, *HI* and the two reactions *IA* and *AB*. To draw the force polygon we select a convenient scale of so many pounds to the inch and, beginning with *bc*, lay off the load line *bc*, *cd*, *de*, *ef*, *fg*, *gh* and *hi*, Fig. 34 (*b*). We have seen that R_2, the right-hand reaction *IA*, is 6,000#, therefore, at the same scale we measure upward 6,000# from point *i* and thus determine point *a*.

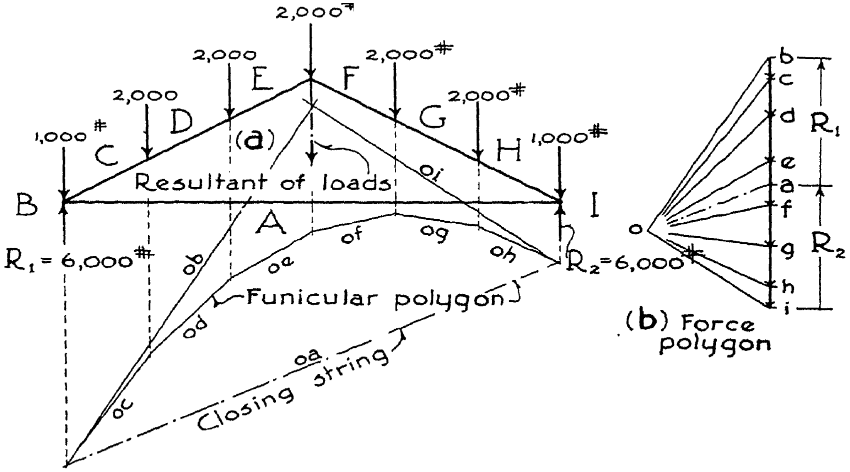


FIG. 34

The next force in order is *AB*, the left-hand support. Since points *a* and *b* are established, this completes the force polygon of the external forces.

For the above elementary problem the solution just given is all that is necessary. The magnitudes of R_1 and R_2 are known by merely inspecting the load diagram. If, however, we wish to determine their magnitudes by graphical methods we can construct a funicular polygon as explained in Art. 26. This polygon is shown directly below the truss. The point *a* in the force polygon, which determines the magnitudes of *IA* and *AB*, shows that each reaction has a magnitude of 6,000#.

Note that the resultant of the loads is *bi*, shown in the force polygon, and that its magnitude is 12,000#. The two rays holding this force in equilibrium are *ob* and *oi*. Hence their intersection in the funicular polygon determines a point through which the resultant of the loads must pass. This is unimportant in this particular problem but very often, particularly for wind loads, it is of great convenience to find this line of action of the resultant.

Problem 56-A. An eight-panel Fink truss, see Fig. 31, has a span of 64′0″ and a total symmetrical vertical load of 32,000#. Draw a force polygon of the external forces, determining the reactions by means of the funicular polygon.

57. **Reactions for Vertical Loads, Unsymmetrical Loading.** The outline of a six-panel truss with web members omitted is shown

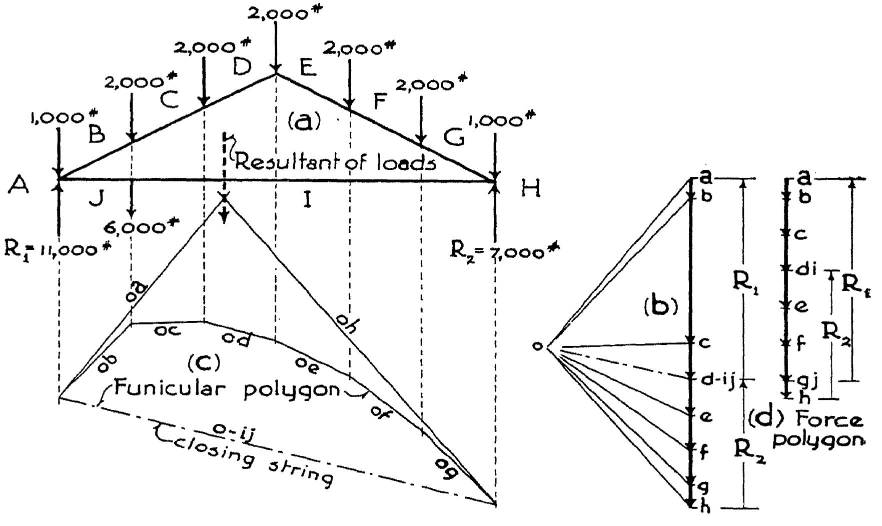


Fig. 35

in Fig. 35 (*a*). A suspended load, *IJ* of 6,000#, results in unsymmetrical loading and consequently the reactions, although vertical, will not be equal in magnitude.

A convenient method of finding the magnitudes of R_1 and R_2 is by constructing a funicular polygon. Note that *BC* and *IJ* have the same line of action. Therefore, for the purpose of drawing the funicular polygon, we may consider the force at *BC* to be 2,000 + 6,000 or 8,000#. The force polygon used in constructing the funicular polygon is shown in Fig. 35 (*b*). First draw *ab*, a downward force of 1,000#. The next force is *BC*, 8,000#. This is followed by *CD*, *DE*, *EF*, *FG*, each of which is 2,000#, and finally *GH*, a force of 1,000#. This completes the load line. As the 6,000# load is combined with *BC*, the space between the supports and below the truss may be called temporarily "*IJ*" instead of the two separate letters. This then results in calling the right reaction *H-IJ* and the left reaction *IJ-A*. The pole *o* is selected and the funicular polygon, Fig. 35 (*c*), is constructed as has been explained. A line drawn from *o* parallel to the closing string determines the point *ij* on the force polygon and hence establishes the magnitudes of R_1 and R_2. The sum of all the loads is 12,000 + 6,000 or 18,000# and is represented by *ah* in Fig. 35 (*b*). By measuring the lengths of *h-ij* and *ij-a*, in this force polygon, we find R_2 = 7,000# and R_1 = 11,000#.

Although it is not required for this particular problem, note that the intersection of lines *oa* and *oh* in the funicular polygon determines the line of action of the resultant of the loads.

We have now determined the magnitudes of the reactions, but Fig. 35 (*b*) cannot be used as a force polygon of external forces to be used in drawing a stress diagram. However, to construct a true force polygon is a simple matter now that all the forces are known. It is shown in Fig. 35 (*d*). Begin with *ab*, a downward force of 1,000#, and follow with *bc*, *cd*, *de*, *ef*, *fg*, each 2,000# in magnitude and finally *gh*, 1,000#. These are the loads on the upper chord. Next comes *HI* an upward force of 7,000# found in Fig. 35 (*b*), thus determining point *i*. The next force is *IJ*, the downward force of 6,000#, and finally *ja* the left reaction, 11,000#, completes the true force polygon of the external forces. Since this is a system of parallel forces, the sides of the force polygon overlap.

As previously stated, it is quite important that the reactions should be determined exactly. The accuracy of the results found by means of the graphical method depends entirely upon the accuracy with which the drawings are made.

Frequently it is desirable to compute R_1 and R_2 by mathematics, avoiding the construction of the funicular polygon. To review the principle of moments to be employed, see Arts. 35 and 36. Since no dimensions are given for this truss, assume the length between supports to be 60′0″. As the loads are equally spaced horizontally, the distances between them will be 10′0″ in each instance. Although we have assumed a length of 60′0″, any length would answer our purpose but it must be divided into six equal parts to determine the relative positions of the loads. First, writing an equation of moments about the right reaction,

$$R_1 \times 60 = (1{,}000 \times 60) + (8{,}000 \times 50) + (2{,}000 \times 40) + (2{,}000 \times 30) + (2{,}000 \times 20) + (2{,}000 \times 10)$$

or

$$60 \times R_1 = 660{,}000 \quad \text{and} \quad R_1 = 11{,}000\#$$

Notice that the resultant of the loads on the upper chord is 12,000# and its resultant line of action is at the center line of the truss. We might have shortened the mathematics if we had observed this in the first instance for, writing an equation of moments about R_2,

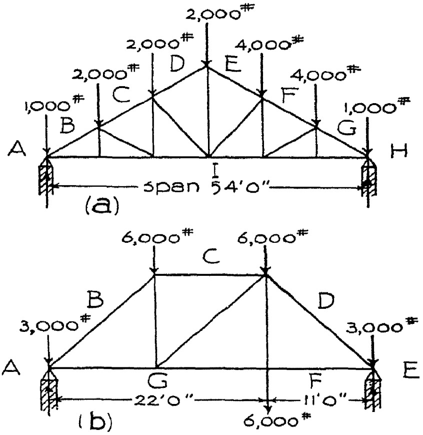


FIG. 36

$$60 \times R_1 = (6{,}000 \times 50) + (12{,}000 \times 30)$$
$$60 \times R_1 = 660{,}000 \quad \text{and} \quad R_1 = 11{,}000\#$$

In a similar manner we find $R_2 = 7{,}000\#$.

PROBLEMS 57-A-B. For the trusses and loads shown in Figs. 36 (*a*) and (*b*) draw the force polygons of external forces by means of the funicular polygon and check the reactions by mathematics.

58. **Wind-Load Reactions.** The reactions due to wind loads on a truss are not vertical. Their magnitudes and directions depend upon the manner in which the truss is connected at its supports. For trusses having spans of approximately 40′0″ and less, and for the usual timber trusses, the truss ends are usually "fixed," rigidly secured to the supports by anchor bolts. This restraint prevents horizontal motion.

For such trusses the directions of the reactions are indeterminate. The exact vertical components, however, may be found but the degree of horizontal resistance is unknown. For trusses having relatively small pitches it is safe to assume that the reactions are parallel to the normal component of the wind, but for steep pitches a condition more likely to exist is that the horizontal components of the reactions are equal. Fig. 37 indicates the supports on masonry walls of trusses having comparatively short spans. Such trusses are secured to the masonry with anchors, and are said to be "fixed." The usual construction for steel trusses is to rivet a *sole plate* to the two angles at the end of the truss, the sole plate resting on the bearing plate. The size of the bearing plate depends upon the load transferred from the truss and upon the allowable bearing strength of the masonry. The computations required in computing its dimensions are similar to those used in designing bearing plates for beams,* and an illustrative problem is given in Art. 123.

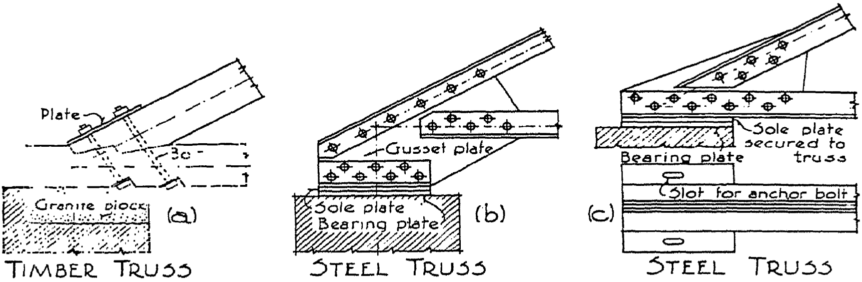


FIG. 37

Because of the expansion and contraction due to temperature variations, it is customary to provide a surfaced base plate under one end of trusses having spans of about 45′0″ and over, the opposite end being anchored to the support. This construction permits the truss to move horizontally at the free end when the length changes with temperature or stress deformation. If such a connection were frictionless the reaction at this end would be vertical due to wind loads, and the tendency for the truss to move horizontally would be entirely resisted by the fixed end. However, such a bearing does offer horizontal resistance equal to the degree of friction that is always present.

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* See Art. 65, *Simplified Engineering for Architects and Builders.*

To determine the allowance to be made for the change in length of steel trusses, an approximate value is obtained by allowing ⅛″ for each 10′0″ of span.

When the span of a truss exceeds 100′0″, the surfaced plates are inadequate and rollers are used. It is customary to use the term "roller bearing" when speaking of trusses of average length having plates that permit lateral motion. The important points to remember are that this type of bearing is used to permit horizontal motion due to possible changes in length of a truss, that the opposite end is restrained and therefore resists the horizontal component due to wind loads and that rollers result in vertical reactions.

59. **Wind-Load Reactions—Fixed Ends—Reactions Parallel.** A triangular truss with wind loads is indicated in Fig. 38 (*a*). *Assuming that the reactions are parallel to the direction of the wind*, let it be required to draw the force polygon of the external forces. Note that the external forces consist of the loads, *AB*, *BC*, *CD*, *DE*, *EF* and the two reactions *FG* and *GA*. The truss is, of course, loaded unsymmetrically and, although we assume the direction of the reactions to be parallel to the wind and know points in their lines of action, their magnitudes as yet are unknown. First, lay off the load line beginning with *ab*, Fig. 38 (*b*). Completed it is *ab*, *bc*, *cd*, *de* and *ef*. Now one method of determining the reactions, though not the simplest, is to draw the funicular polygon as shown at Fig. 38 (*c*). A line drawn from point *o*, Fig. 38 (*b*), parallel to the closing string determines the point *g* and consequently *FG* and *GA* are established. Now all that need be done to find the magnitudes of the reactions is to scale the lengths of *fg* and *ga*.

Here is a much simpler method of accomplishing the same results found above. First draw the load line *ab*, *bc*, *cd*, *de* and *ef*, Fig. 38 (*d*). The resultant of the wind loads is *AF*, 8,000#. Since the wind loads on the side of the truss are placed symmetrically it is obvious that their resultant lies at the midpoint, or on the same line of action as *CD*. This was verified by the funicular polygon, Fig. 38 (*c*). The problem is now simplified for we may say that there are but three external forces, the wind-load resultant *AF* and the two reactions *FG* and *GA*. It can be shown that the magnitudes of the reactions are in the same proportion as the two divisions of the lower chord cut by the wind-load resultant. These divisions are marked *x* and *y*, Fig. 38 (*a*). All that remains, therefore, is to divide the load line in the same proportion as *x* is to *y*. To accomplish this, draw any line from *f*, as *fm*, and lay off divisions *fn* and *nm* proportional to *x* and *y*. A line drawn from *n* to the load line, parallel to *ma*, divides the load line into the desired proportion, hence establishing *fg* and *ga*, the two reactions. In using this convenient method it is necessary to inspect the truss diagram to see which reaction is the greater so that the divisions *x* and *y* may be properly located with respect to the load line. In this particular instance it is noted that since the resultant wind

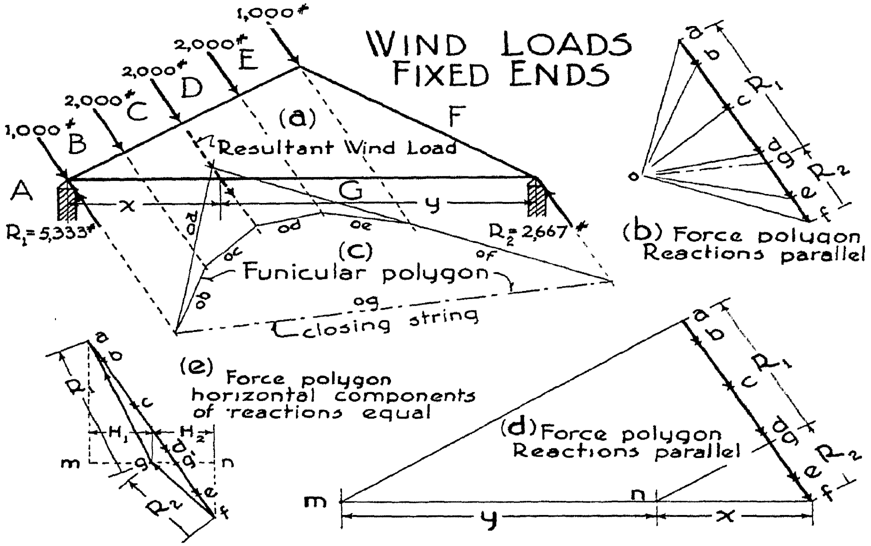


FIG. 38

load is nearer R_1 than R_2, R_1 or GA will have the greater magnitude and consequently x and y are located in this manner with respect to the load line.

Finding the reactions by this *proportional line method* may be accomplished very quickly indeed. Note that the results agree with those found by using the funicular polygon in Fig. 38 (*b*).

Horizontal Components Equal. The assumption in the previous example was that the two reactions are parallel. The other possibility, as explained in Art. 58, is that the horizontal components of the reactions are equal. This is quite simple.

Draw the load line *ab*, *bc*, *cd*, *de* and *ef*, Fig. 38 (*e*), and determine the reactions, assuming them to be parallel as has just been explained. This establishes the point g'. Now the horizontal component of the wind is simply the horizontal distance between points a and f. Therefore, draw a horizontal line through g' and vertical lines through points a and f. The intersections, m and n, determine the horizontal component of the wind load. Next divide mn into two equal parts, mg and gn, *the two equal horizontal components*. R_2 therefore will be fg, and R_1 will be ga, not necessarily parallel. Attention should be called to the fact that the vertical components of the reactions remain unchanged in this construction.

For the usual trusses in practice, stresses in truss members found in accordance with either of the two above assumptions do not differ greatly. Therefore, the assumption that the reactions are parallel may be accepted for common trusses with pitches of less than about 30°. However, for roofs having greater pitches, the assumption of equal horizontal components is more nearly in accord with actual conditions.

To find the magnitude of the reactions by mathematics is remarkably easy. The sum of the reactions, assuming them to be parallel, must, of course, equal the resultant wind load. We know also that this resultant wind load will be proportioned to R_1 and R_2 in the same proportion as x and y. Therefore

$$x + y \times \text{resultant wind load}$$

also

$$\times \text{resultant wind load}$$

For the purpose of illustration, suppose the span for the truss indicated in Fig. 38 (*a*) is 60′0″ and that $x = 20'0''$ and $y = 40'0''$. The resultant wind load is 8,000#. Then

$$\frac{40}{60} \times 8{,}000 = 5{,}333.3\#$$

and

$$R_2 = \frac{20}{60} \times 8{,}000 = 2{,}666.6\#$$

The above discussion has included several methods of finding the magnitude of the reactions. It will be well to become acquainted with them all so that any one may be employed as circumstances require.

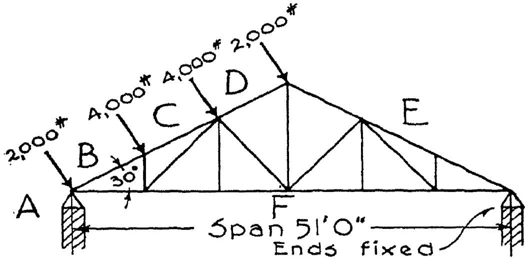


FIG. 39

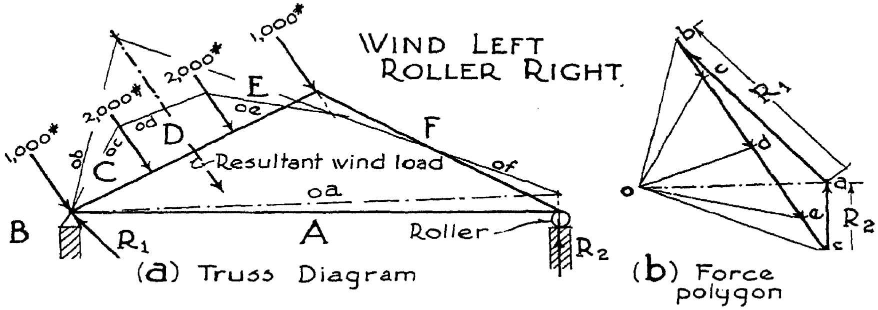
PROBLEM 59-A. Fig. 39 shows a truss having a span of 51′0″ and a pitch of 30°. The truss is "fixed" at the supports and a total wind load of 12,000# is shown. Let it be required to construct a force polygon of external forces,

(a) assuming the reactions parallel, using the funicular polygon
(b) assuming the reactions parallel, using the "proportional line" method
(c) assuming the reactions parallel, computing reactions by mathematics
(d) assuming the horizontal components of the reactions to be equal

60. Wind-Load Reactions—Roller Under One End. The end connections for the previous trusses have been fixed, that is, restrained against horizontal action. Now let us investigate reactions for trusses having one end supported on a roller. Since, theoretically, such a support is incapable of offering horizontal resistance, the reaction at this end must be vertical and the tendency to move horizontally will be resisted by the fixed end.

Fig. 40 (*a*) indicates in outline a six-panel truss with wind loads on the left and a roller at the right support, the left support being fixed. By data we know the direction, lines of action and magnitude of the wind loads. We also know the line of action of R_2, but all that is known of R_1 is a point in its line of action, namely, the support. Let it be required to draw a force polygon of the external forces.

One method of accomplishing this is by constructing a funicular polygon. First draw the load line beginning with *bc* and continue with *cd*, *de* and *ef*, Fig. 40 (*b*). *FA*, the next force, is the right reaction and this we know to be vertical. Therefore, through *f* extend a vertical line. The point *a* will lie somewhere on this line and when it is found we will have determined *fa* and *ab*, the two reactions. Next select a pole *o* and draw the rays. Since the only point we know on the line of action of *AB* is the point of support, we must begin the funicular polygon here. Therefore, through this point draw *oc*, the side of the funicular polygon, and continue with the remaining sides as previously described. A line through *o* parallel to the closing string intersects the vertical line through *f* at *a*, Fig. 40 (*b*), thus establishing *fa*, the right reaction. A line drawn from *a* to *b* determines *AB*, the left reaction, in both magnitude and direction.



(a) Truss Diagram (b) Force polygon

FIG. 40

The intersection of the sides of the funicular polygon *of* and *ob* gives us a point through which the resultant of the wind load must pass. This resultant is not used in the method just explained, but often it is convenient to know its position.

61. **Roller Under One End—A Shorter Solution.** The previous method of determining the reactions when a roller is employed

required the construction of a funicular polygon. When the loads are symmetrical, as they most frequently are, the work may be simplified by the following procedure.

Again in Fig. 41 (*a*) we have a diagram representing wind loads on the left side of a truss with a roller at the right. Our problem is to draw the force polygon of the external forces. Fundamentally, the system consists of three forces, the wind load and the two reactions. By data they are in equilibrium. Since *FA* is vertical, the forces are not parallel and therefore must meet in a common point, as we know that any system of three forces, not parallel and in equilibrium, must be concurrent. See Art. 22. To find the resultant of the wind loads we can construct a funicular polygon as shown. This would be necessary if the loading was unsymmetrical but in this instance, *as is generally the case*, we can locate its position by observation, namely at the midpoint of the loads. Therefore all that is necessary is to continue the line of action of the resultant until it meets the line of action of *FA*, at *m*. This gives us the point in common of the three external forces. Consequently a line drawn from *m* to the left-hand support determines the *direction* of *AB*.

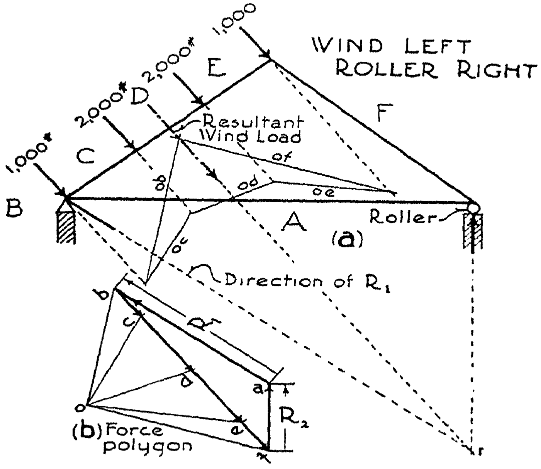


FIG. 41

To draw the force polygon we begin with *bc* and lay off the load line *bc*, *cd*, *de* and *ef*, Fig. 41 (*b*). From *f* draw a vertical line; point *a* will lie somewhere on this line. As we have determined the direction of *AB*, we now draw a line through *b* parallel to the direction of *AB* found in Fig. 41 (*a*); the point *a* will occur on this line. Therefore, the intersection of this line and the vertical line through *f* determines the point *a* and consequently the two reactions *FA* and *AB*.

For the wind coming from the right, with a roller at the right support, a similar construction is shown for determining the reactions in Figs. 42 (*a*) and (*b*). Again the resultant is found by the funicular polygon, a step not required for this loading, for obviously its line of action is at the center point of the upper chord. *The important principle involved in finding reactions by this method is that three forces, not parallel and in equilibrium, must meet in a common point.* By applying this principle, the direction of the unknown reaction is established, thus permitting the completion of the force polygon.

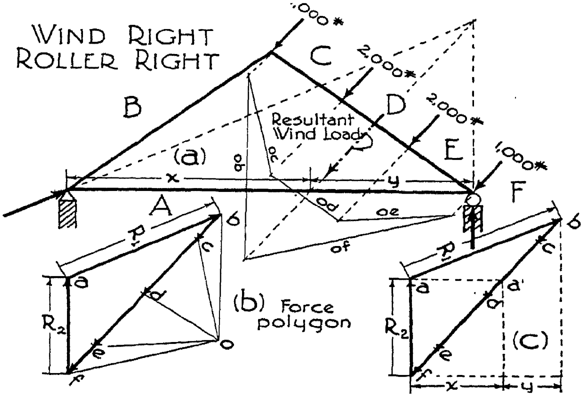


FIG. 42

Suppose we had been given the truss shown in Fig. 42 (*a*) with the stipulation that the truss had fixed ends. By the principle of the proportional line, *fa'* and *a'b* are quickly found, Fig. 42 (*c*), thus giving us the two reactions but with the assumption that the reactions are parallel. Now if a roller is substituted at the right reaction, the supporting force can no longer be *fa'*; it can only be its vertical component. Consequently if we draw a horizontal line through a' to intersect a vertical line through f, their intersection gives us the desired vertical component and therefore the two reactions FA and AB are found. Note that this is the same result found in Fig. 42 (*b*).

PROBLEM 61-A. A Fink truss having a pitch of 30° and a span of 66′0″ is subjected to wind loads as shown in Fig. 43. A roller occurs at the right support. Draw the force polygon of external forces.

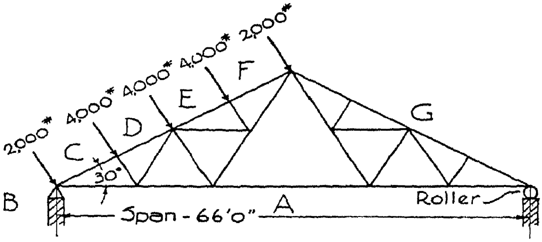


FIG. 43

PROBLEM 61-B. For the truss shown in Fig. 43 assume the wind to come from the right, reletter the truss and draw the force polygon of external forces.

62. **Resultant Reactions.** To find the pressure exerted by a truss on its supports, it is necessary to find the resultant reactions. As an illustration, Fig. 44 represents a six-panel truss subjected to a total vertical load of 24,000# and a total wind load of 9,000#. Since the truss is symmetrically loaded *with respect to the vertical loads*, it is obvious that each reaction will be equal to one-half of the total vertical load or 12,000#. Their directions, of course, are vertical and the reactions are drawn to scale at the supports.

Assuming the truss to be fixed at both ends and that the reactions due to the wind loads are parallel, the wind-load reactions are found by the proportional line method to be R_1 = 6,000# and R_2 = 3,000#. These reactions likewise are drawn to scale at the supports. The true or resultant reactions, due to the vertical loads and wind loads acting simultaneously, may be readily found in direction and magnitude by completing the parallelogram of forces as shown. By scaling the resultant reactions, R_1 is found to be 17,500# and R_2 equals 14,500#. It should be noted that the wind is shown coming from the left in Fig. 44, resulting, in this instance, in R_1 being greater in magnitude than R_2. However, if the direction of the wind changes and comes from the right, R_2 becomes the greater reaction. The customary procedure is to assume the wind to come from either the right or left and to use for each reaction whichever force is the greater magnitude.

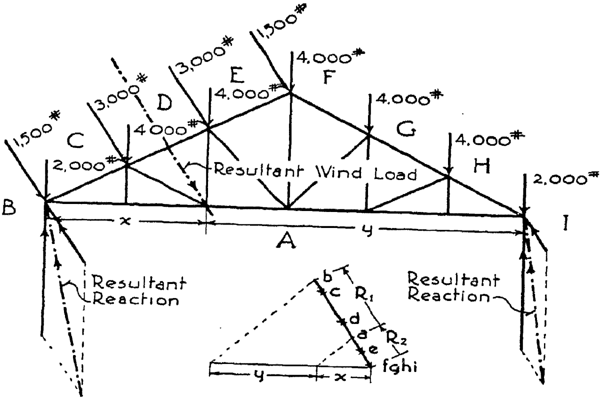


FIG. 44

CHAPTER 8

STRESSES IN ROOF TRUSSES

63. Stresses in Truss Members. Most of the trusses in common use have external forces, or loads, applied only at the panel points. When this condition exists, the combined effect of the loads and reactions is to stress the truss members by tending to lengthen some and to shorten others. By this is meant that certain members receive tensile stresses while others are subjected to compression. We use the word *character* of a stress to denote its kind, tension or compression.

Theoretically, due to the rigidity resulting from the connections of members at joints, and also because of resulting deformations and distortions, stresses other than compression and tension are developed. However, since these stresses are relatively small in comparison to the major stresses, they may be ignored in the design of usual trusses. It is of importance, therefore, to have this point clearly in mind. When the loads are applied at the joints, the stresses developed in the members are either tension or compression. The tensile members are known as *ties* while the members resisting compression are *struts,* actually columns.

A *beam* may be defined as a structural member subjected to transverse forces which tend to *bend* rather than shorten or lengthen it. Bear in mind that the members of a truss are rarely beams; they are primarily ties or struts. Of course, the weight alone of a member tends to bend it, but this stress due to bending is so small it may be ignored in computations. Again, if purlins occur between joints, the upper chord is subjected to bending as well as compression. This, however, is a special case and, unless attention is particularly called to it, the only stresses considered in truss members will be tension and compression.

In order to design a truss it is necessary to determine the *character* of the stresses in the various members and also their *magnitudes.* By magnitude is meant the number of units of force the member is required to resist. For instance, we say a certain member resists a tensile force of 25,000#, or a strut 8′6″ in length is subjected to an axial load of 30,000#.

64. **Graphical Analysis.** There are different methods that may be employed to find the character and magnitudes of stresses in truss members. Briefly, one is by mathematics and the other is graphically. The graphical method is usually the most convenient and will be used in the solution of the problems in this book. It has the advantage of affording a rapid solution and, above all, provides a check on the work.

Fundamentally, the graphical solution consists in nothing more than the application of the principle of the force polygon as applied to the forces concurrent at the joints. Before proceeding further, review Art. 21 and note how readily the stresses in three concurrent members are found when only one force is given. At any joint in a truss we have the condition of a number of concurrent forces in equilibrium. By the principles previously discussed, we know the force polygon for these forces must close and this principle enables us to complete the force polygon and hence determine the stresses in the members. *Regardless of the number of forces at a joint the force polygon may be drawn provided not more than two of the forces are unknown.*

Before beginning the construction of a stress diagram for a truss, it is essential that all the external forces be determined. The loads and the reactions constitute the external forces and they may be found by the methods previously explained. If the truss is symmetrically loaded and the forces are vertical, the reactions are likewise vertical and each reaction is equal to one-half the total load. If the loads are oblique with the vertical, such as wind loads for example, or if the vertical loading is unsymmetrical, the reactions may be computed by mathematics or by the construction of the funicular polygon. These various methods have been explained in Chapter 7.

Now let us proceed with a truss and discover how extremely simple it is to construct a stress diagram.

65. **Construction of a Stress Diagram.** A truss having a total vertical load of 12,000# is represented in Fig. 45 (*a*). The truss is symmetrically loaded. Let it be required to construct a stress dia-

gram for this truss in order to determine the magnitude of the stresses within the truss members.

First of all determine the reactions. The total load is 12,000# and, as the truss is symmetrically loaded, each reaction is equal

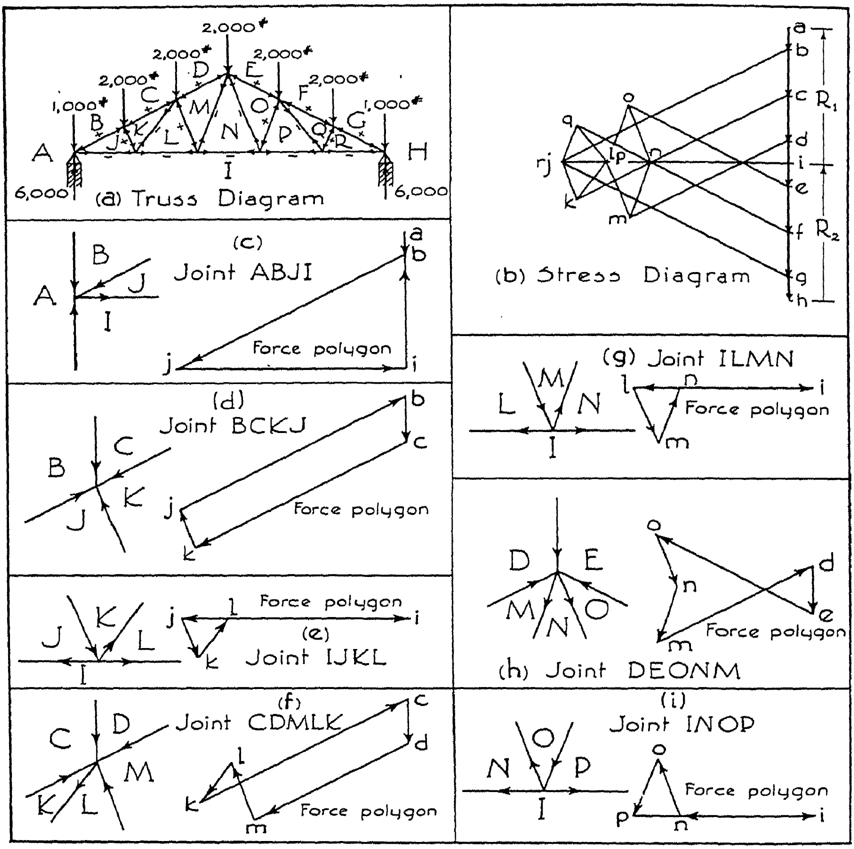


FIG. 45

to ½ × 12,000 or 6,000#. The reactions are vertical. We now know all the external loads.

In accordance with Art. 20, a clockwise direction will be employed in identifying the various forces. To begin, first consider the forces about the joint *ABJI*. There are four forces here, *AB*, *BJ*, *JI* and *IA*, and of them we know two, *IA* = 6,000# and *AB* = 1,000#. Since by data the four concurrent forces are in equilibrium, their force polygon must close. To construct this force polygon, erect a vertical line *ia* at any convenient scale, as, for instance, 1″ = 4,000#. Thus *ia* in the force polygon, Fig. 45 (*c*), will be 1½″ in length. Note that since the force *IA* is upward, *i* lies at the bottom and *a* at the top of the line *ia*. The next force in order is *AB* which we know to be 1,000#. Therefore, from the point *a*, on the line *ia* just drawn, measure downward 1,000#, or ¼″, to establish the point *b*. Continuing in a clockwise direction, the next force is *BJ*. Of this force we know only its direction; its magnitude is unknown. Therefore through *b* draw a line parallel to *BJ*; the point *j* will occur at some position on this line. The next force is *JI*. Through point *i* in the force polygon, draw a line parallel to *JI*. The point *j* will lie somewhere on this line. Since *j* is also on the line through *b* parallel to *BJ*, the intersection of these two lines determines the point *j* and thus completes the force polygon for the forces *IA*, *AB*, *BJ* and *JI*. The force polygon may be read *abjia*. Now to determine the *magnitude* of the stresses in members *BJ* and *JI*, all that is necessary is to measure the lengths of the lines *bj* and *ji in the force polygon* at the same scale used in laying off the forces *IA* and *AB*. For these members, *BJ* = 10,000# and *JI* = 8,660#. Note particularly that *the lengths of the members in the truss are not an indication of the magnitudes of the stresses; the magnitudes are represented by the length of the corresponding sides in the force polygon.*

As yet it is impossible to draw the force polygon for the forces about the joint *IJKL*, for of the four forces only *IJ* has been found, there being three unknowns. No matter how many forces there are, there must not be more than two unknowns. Therefore let us try the joint *BCKJ*.

About the joint *BCKJ* there are four forces, *BC*, *CK*, *KJ* and *JB*. *JB* we have found to be 10,000# and *BC* is 2,000#. We note that here there are only two unknowns, *CK* and *KJ*, hence it is possible to construct a force polygon. At the same scale used previously, draw *jb* and *bc*, Fig. 45 (*d*), two sides of the force polygon. Next draw a line through *c* parallel to *CK* and a line through *j* parallel to *KJ*. Since the point *k* lies at some point on each of these two lines it must be at their point of intersection. Thus point *k* is established and the force polygon is completed. It is read *jbckj*. The magnitudes of the stresses in members *CK* and *KJ* are found by measuring their corresponding lengths in the force polygon just completed.

We may now analyze joint *IJKL* since *JK* has been found and there are now but two unknowns. Therefore draw *ij* and *jk*, the two known sides of the force polygon, Fig. 45 (*e*). Through *k* draw a line parallel to *KL* and through *i* a line parallel to *LI*. The point *l* lies at their point of intersection and the force polygon is completed, *ijkli*.

In the same manner we can construct force polygons for each of the remaining joints of the truss. Fig. 45 (*f*), (*g*), (*h*) and (*i*) show the force polygons for joints *CDMLK*, *ILMN*, *DEONM* and *INOP*. The remaining joints are similarly constructed.

Since these individual force polygons determine the stresses within the truss members, they may be called *stress polygons*. In the analysis of a truss by the graphical method just explained, considerable effort may be saved if the various polygons are combined into one diagram and this procedure is always followed. This diagram is called the *stress diagram*. It is constructed in the following manner.

To construct a stress diagram, the very first step is to draw a force polygon of the external forces. Reading in a clockwise manner, these forces consist of *AB*, *BC*, *CD*, *DE*, *EF*, *FG*, *GH*, *HI* and *IA*. To begin, lay off the *load line ab*, *bc*, *cd*, *de*, *ef*, *fg* and *gh*, Fig. 45 (*b*). This line is vertical, parallel to the direction of the loads. The next force is *HI*, an upward force equal to one-half the total load on the truss, thus establishing the point *i*. The last force is *IA*, also upward, completing the force polygon of the external forces.

Now, beginning with the joint *ABJI*, we have *ia* and *ab*, so through point *b* we draw a line parallel to the truss member *BJ*, and through point *i* draw a line parallel to *JI*. The intersection of these two lines establishes point *j*, completing the force polygon for the forces about joint *ABJI*. Note that this polygon is identical to the polygon found in Fig. 45 (*c*). We continue in the same manner with the succeeding joints and have finally the complete stress diagram. The last point found is *r*, and, if the drawing is made accurately, the line connecting points *r* and *g* will be parallel to the member *RG.* If these lines are not parallel, the stress diagram is incorrect; an error has been made at some point. This self-checking is one of the great advantages in analyzing trusses by the graphical method.

66. **Character of Stresses.** The preceding article explained the construction of the stress diagram and the method of finding the magnitude of the stresses in the members. As well as knowing the magnitude of the various stresses, it is equally important that we know their *character*, that is, whether they be in tension or compression. This may readily be accomplished by the following steps.

STEP 1. Select a *reference joint* in the truss diagram and identify a particular member. Care must be exercised to read the member clockwise or counter-clockwise, whichever direction was used in drawing the stress diagram. A clockwise direction is used throughout this text.

STEP 2. Refer to the stress diagram and read this stress, *the letters being in the same sequence as read in Step 1.* Note the direction of this stress in accordance with this sequence, such as left to right, downward to left, upward, etc.

STEP 3. Return to the truss diagram and, *with respect to the reference joint* selected in Step 1, note the *direction* of the force determined in Step 2. If the direction reads toward the reference joint the stress is compression; if it reads away from the joint the stress is tensile.

Let us apply this procedure to a particular member, for instance, the part of the upper chord adjacent to the left support, Fig. 45 (*a*).

STEP 1. Consider the member *BJ* about the joint *ABJI.* Note that this member is *BJ*, *not JB*, because the stress diagram was drawn in a clockwise manner.

STEP 2. Referring to the stress diagram, Fig. 45 (*b*), *bj* reads downward to the left.

STEP 3. Returning to the truss diagram, Fig. 45 (*a*), the member *BJ* read downward to the left reads *toward* the reference joint *ABJI* and therefore the member is in compression.

Suppose we had considered this member with reference to its other end, the joint *BCKJ.*

STEP 1. With reference to the joint *BCKJ* this member is read *JB*, clockwise direction.

STEP 2. In the stress diagram *jb* reads upward to the right.

STEP 3. Reading *JB* upward to the right, with respect to joint *BCKJ*, we read toward the joint. Consequently the member is in compression. Notice that this is in agreement with the stress found by taking the joint *ABJI* as a reference joint. It makes no difference, therefore, which end of a member is considered, but the sequence of the letters used to identify the member is, of course, vital.

Let us try one more member.

STEP 1. With respect to the reference joint *NOPI*, consider the member *IN*.

STEP 2. In the stress diagram *in* reads from right to left.

STEP 3. Reading *IN* in the truss diagram from right to left we read away from the reference joint *NOPI*, hence the member *IN* is in tension.

In this treatise a member in compression will be indicated with the plus sign (+), and tension members with the minus sign (−). Some authorities reverse these signs. Still another method of designating the character of stresses is to place arrows on the ends of members near the joints. If the stress is compressive the arrows point toward the joints, and away from the joints if the stress is tension. These arrows are shown on the truss diagram, Fig. 45 (*a*), as well as the plus and minus signs.

67. Character of Stresses by Inspection. By adopting the procedure just explained, the characters of the stresses are readily found provided the stress diagram has been drawn. For the simpler trusses it is often possible to analyze the stresses merely by inspection of the truss diagram. In doing this one must bear in mind that compressive forces due to vertical or wind loads are applied at the joints of the upper chord. For the usual truss supported at its ends, the upper chord is in compression and the lower chord is in tension.

Refer to Fig. 45 (*a*) and consider the members about the joint *BCKJ*. We know that *JB* and *CK*, being upper chord members, are in compression and that *BC*, an external force, is also a compressive force at this joint. Now remembering that the four forces are in equilibrium, and that the point *BCKJ* will not change its position under the action of the four forces, *KJ* must be in compression, acting *toward* the joint; otherwise the force *BC* would tend to move the joint downward.

Again, inspect the joint *JKLI*. We have found *JK* to be in compression; *LI* and *IJ* are in tension since they are members of the lower chord. To hold this joint in equilibrium *KL* must be in tension to neutralize the downward action of *JK*. Usually the web members alternate in compression and tension up to the center of the truss. Remember also that, for symmetrical loading, the stresses in the members in each half of the truss are similar. For instance, if *KL* has been found to be in tension, *PQ*, which has a similar position in the right side of the truss, will likewise be in tension.

In determining the character of stresses by inspection, it is necessary to begin at a point at which there is but one unknown. In Fig. 45 (*a*) the joints may be taken in the following sequence, *BCKJ*, *JKLI*, *CDMLK* and *LMNI*.

PROBLEM 67-A. Draw the stress diagram for the fan truss shown in Fig. 46. Mark on the truss members the character and magnitude of the stresses.

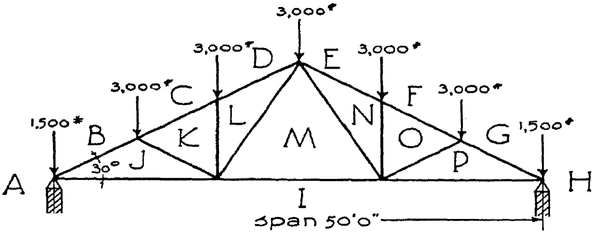


FIG. 46

68. **Pratt Truss.** Two commonly used roof trusses are the Pratt and Howe trusses, Figs. 47 and 48. They differ in the direction of the diagonal web members. Both trusses are shown as having six panels but any number of panels may be used, depending, of course, on the length of the span. Fig. 47 (*a*) represents a Pratt truss with vertical loads. Since the diagonal members are in tension and are relatively long, this truss is better adapted to steel construction than timber.

The construction of the stress diagram presents no difficulties. The total load is 18,000# and, as the truss is symmetrical and is symmetrically loaded, each reaction is vertical and has a magnitude of 18,000 × ½ or 9,000#.

First draw the force polygon of the external forces. These forces are all known. Draw *bc*, *cd*, *de*, *ef*, *fg*, *gh* and *hi*, the loads, Fig. 47 (*b*). The line thus drawn is known as the *load line*. The next force in order is *IA*, an upward force of 9,000#, thus determining the point *a*. An upward force of 9,000#, *ab*, completes the force polygon of the external forces.

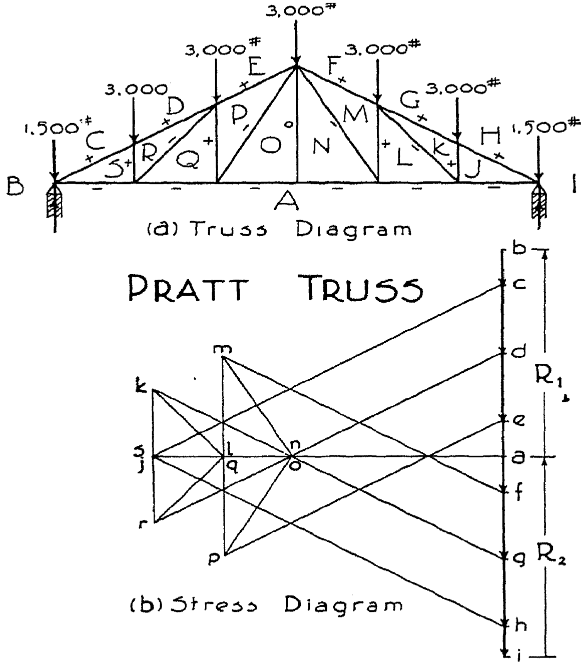


FIG. 47

The only joints at which we might begin to construct the stress diagram are *BCSA* or *HIAJ*, the joints at the supports. At each of these joints there are four concurrent forces two of which are known, two being unknown. First let us consider *BCSA*. In connection with the load line just drawn, draw a line through *c* parallel to the member *CS*. The point *s* will lie at some position on this line. The next member in order is *SA*, therefore through point *a* draw a line parallel to member *SA*. The point *s* lies somewhere on this line and since it also occurs on the line through *c* parallel to *CS*, it must be at their point of intersection.

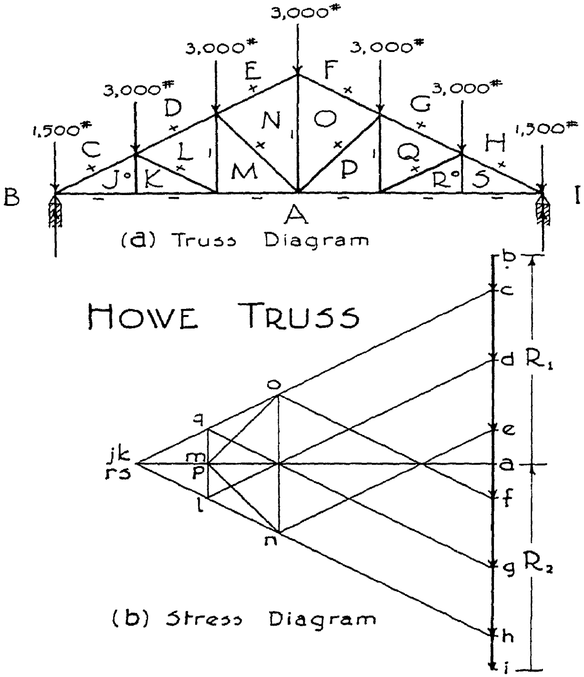
The joint *SRQA* may not be considered next, for at this point there are four forces and as yet only one, *AS*, is known. Therefore take joint *CDRS*; here there are only two unknowns. A line through *d* parallel to *DR* and a line through *s* parallel to *RS* intersect at a point which is *r*. In a similar manner we may take the remaining joints in the truss and thus complete the stress diagram as shown in Fig. 47 (*b*).

On inspecting the stress diagram it is noted that the letters *n* and *o* occur at the same point. This indicates that, since *on* has no length, the member *ON* has a zero stress. This is called a *redundant member* and is only employed to prevent a possible sag in this portion of the lower chord. For the stress diagram just drawn, only loads on the upper chord were considered, but if a ceiling had been suspended from the lower chord, a tensile stress would have been developed in member *ON*.

A redundant member may be readily identified by inspecting the truss diagram. At the joint *ONA* three forces are in equilibrium. Since *AO* and *NA* have the same line of action, one of the forces must be the equilibrant of the other since any other single force acting at this joint would disturb the state of equilibrium. Hence the stress in member *ON* must be zero. Whereas the construction of the stress diagram readily indicates a redundant member, it is frequently convenient to identify such members by merely inspecting the truss diagram.

69. **Howe Truss.** Fig. 48 (*a*) represents a Howe truss having vertical loads on the upper chord only. This truss differs from the Pratt truss in the direction of the diagonal web members. It is commonly used in timber construction and, as the vertical members are ties, rods are used since metal is more readily adapted to tensile stresses than timber. Although the lower chord is in tension, it is customary to use timber for these members to facilitate the detailing of the joints. A ceiling suspended at the lower chord likewise suggests the use of timber for these members owing to the convenience in framing.

The stress diagram for this truss is shown in Fig. 48 (*b*). It presents no difficulties in its construction. Note that the two vertical members, *JK* and *RS* are redundant members and may be omitted if there are no suspended loads.

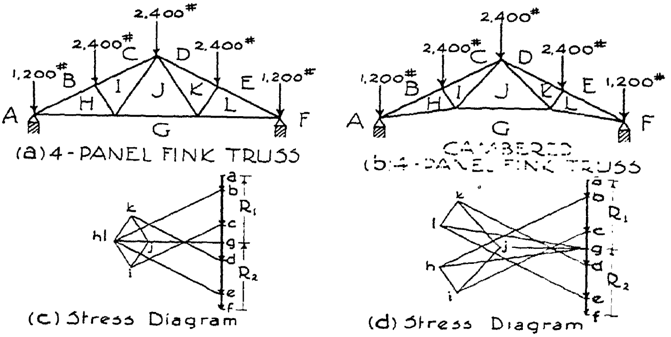


(a) Truss Diagram

(b) Stress Diagram

FIG. 48

70. Cambered Trusses. When a portion of the lower chord of a truss is raised, as shown in Fig. 49 (*b*), the truss is said to be



(a) 4-PANEL FINK TRUSS

(b) 4-PANEL CAMBERED FINK TRUSS

(c) Stress Diagram

(d) Stress Diagram

FIG. 49

cambered. The purpose is to provide greater head-room under the central part of the truss and also, possibly, to enhance its appearance. Trusses having horizontal lower chords, particularly trusses of long spans, often appear to sag, and cambering overcomes this illusion.

Figs. 49 (*a*) and (*b*) represent trusses of similar loads and spans. Their stress diagrams at (*c*) and (*d*) show how cambering increases the stresses in certain members. It is evident from Fig. 49 (*d*) that the stresses increase in magnitude with the degree of camber. The most common practice is to camber the lower chord one-sixth the height of the truss.

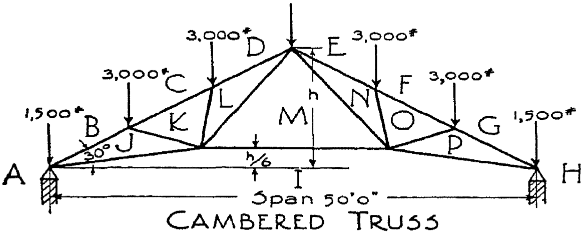


Fig. 50

Problem 70-A. Construct a stress diagram for the cambered truss shown in Fig. 50. Mark on the truss members the character and magnitude of the stresses and compare them with the stresses found in Problem 67-A.

71. **Crane Truss.** Let it be required to determine the stresses in the Crane truss shown in Fig. 51 (*a*). In order to draw the force polygon of the external forces, first consider of what they consist. The load *AB*, 10 tons, is completely known. The resultant pressure at the ground, *BD*, is the resultant of *DC* and *CB*. Thus far we know only a point in its line of action, since *DC* and *CB* are of unknown magnitudes. The remaining external force is *DA*, the equilibrant of the stress in the member *DA*. This force is a tensile force in the earth and its line of action must coincide with the member *DA*. Since the three external forces, *AB*, *BD* and *DA*, are in equilibrium, and are not parallel, they must have a point in common. This point is readily found by extending the lines of action of *AB* and *DA*. They intersect at point x. A line drawn from x to the

point at which *DC* and *CB* meet at the surface of the ground determines the *direction* of the external force *BD*. Thus the directions of the three external forces are now established.

To draw the force polygon of these external forces draw, at a convenient scale, *ab* parallel to the force *AB*, having a length equal to 10 tons, Fig. 51 (*b*). Next draw a line through *b* parallel to the external force *BD* and also a line through point *a* parallel to the external force *DA*. Their intersection establishes point *d*, thus determining the three external forces. In Fig. 51 (*b*), *bd* is shown as a dotted line since it is not a stress in a member of the truss but represents the resultant pressure under the members *DC* and *CB*. To find the stresses in *BC* and *CA*, draw a line through *b* parallel to *BC* and also a line through point *a* parallel to *CA*; their intersection determines point *c*. Connecting points *c* and *d* gives us the stress in the member *CD*. Note that this line must be parallel to member *CD* and if this condition does not exist an error has occurred at some point in constructing the stress diagram.

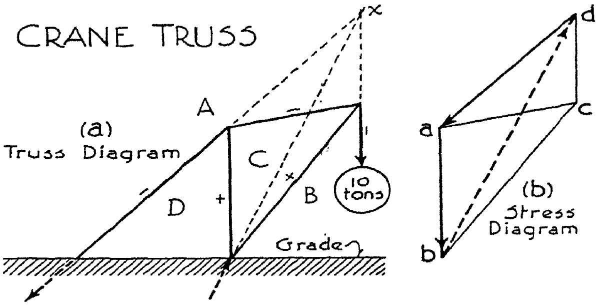


FIG. 51

The magnitudes of the stresses are found by measuring the lengths of the lines in the stress diagram. Note that member *CD* has a greater length than member *AC*, whereas the stress in *CD*, shown in the stress diagram, is smaller than the stress in member *AC*. Again we see that the length of a member is not an indication of its stress; the magnitude of the stresses are represented by the lengths of their respective lines in the stress diagram. Note also that the pressure of the truss on the ground is approximately twice the magnitude of the "pull" exerted by member *DA*.

72. Sawtooth Truss. The sawtooth roof is used principally in industrial buildings, its purpose being to aid in the distribution of daylight over floor areas. The construction consists of a glass area in the steepest portion of the roof, preferably facing the north, the remainder being composed of a suitable roofing surface. In its erection, particular care must be taken to guard against water leakage and the possibility of condensation.

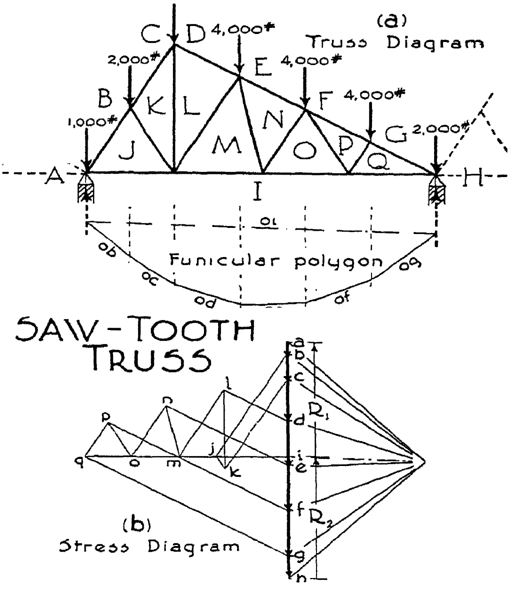


FIG. 52

One type of truss supporting a sawtooth roof is shown in Fig. 52 (*a*). As indicated, this truss is unsymmetrical as well as the loading. To construct the stress diagram, first draw the load line *ab*, *bc*, *cd*, *de*, *ef*, *fg* and *gh*. The reactions *HI* and *IA* are unknown and must be determined in order to complete the polygon of external forces. These reactions may be computed by the principle of moments as explained in Art. 57, or by the construction of a funicular polygon, the latter being shown in Fig. 52. The pole *o* is selected, the rays are drawn and the funicular polygon constructed directly beneath the truss diagram. A line drawn from *o*, parallel to the closing string *oi*, determines the point *i* in the force polygon of external forces, thus establishing the reactions *HI* and *IA*. To draw the stress diagram we may begin at either joint *ABJI* or *GHIQ*. The stress diagram is developed as has been previously described and is shown in Fig. 52 (*b*).

The characters of the stresses are purposely omitted in the truss diagram. Before analyzing these stresses, as explained in Art. 66, try to identify them by inspection, beginning with the compressive stress *PQ*.

73. Grandstand Truss. Another unsymmetrical truss is shown in Fig. 53 (*a*). This is not a simple truss since the left-hand end overhangs R_1, the left reaction. The two reactions not being equal, it is necessary to determine their magnitudes in order to draw the force polygon of external forces. As the loads in this instance are placed at equal distances horizontally, their resultant, 20,000#, acts at 2½ units of distance from R_2, the right reaction. Then, by the principle of moments,

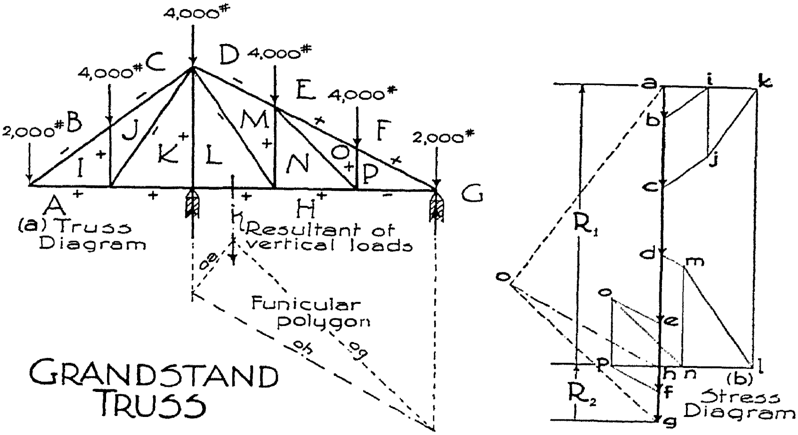


FIG. 53

$$3 \times R_1 = 20,000 \times 2.5$$

$$R_1 = 16,666\tfrac{2}{3}\#$$

and $$R_2 = 20,000 - 16,666\tfrac{2}{3} =$$

Fig. 53 illustrates how readily the funicular polygon may be employed to obtain the same result. Note that the resultant of the loads is AG and that it acts at the midpoint between the ends of the truss. Considering then the three forces AG, GH and HA, the funicular polygon is readily drawn, thus determining point h and consequently R_1 and R_2. Attention is called to the fact that pains must be taken to locate point h accurately, otherwise the stress diagram will not close. The complete stresss diagram is shown in Fig. 53 (*b*). Note the character of stresses in the chord members.

PROBLEM 73-A. Draw the stress diagram for the truss shown in Fig. 54, determining the reactions by mathematics as well as by the funicular polygon, and mark the character and magnitude of the stresses on the truss members.

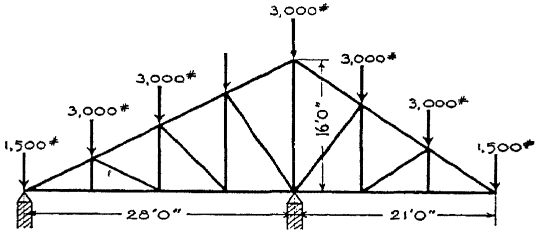
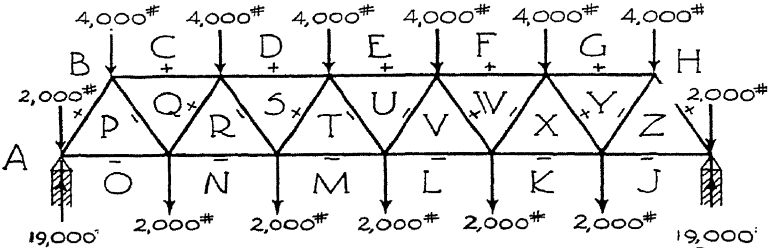


FIG. 54

74. Warren Truss—Suspended Loads.

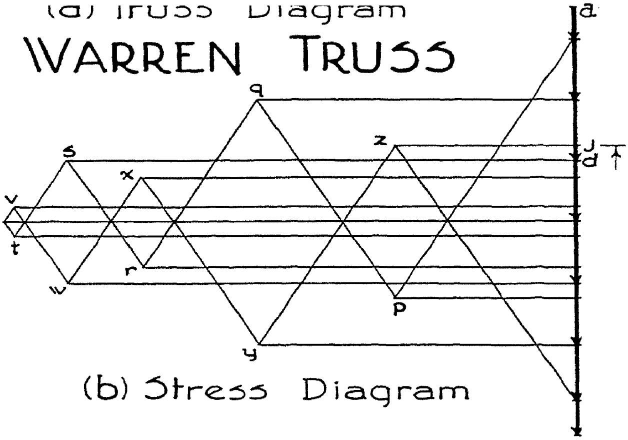
Trusses with parallel chords are often used for flat roofs or for floor supports instead of plate girders. The advantage of the truss over the plate girder lies in the fact that the open spaces between web members permit the placing of pipes and ducts. For roofs, the upper chord may be inclined slightly, if desired, to provide for drainage. A depth of one-fifth or one-sixth of the span generally proves to be most economical.

A Warren truss is shown in Fig. 55 (*a*). In addition to a load of 28,000# on the upper chord, there is a suspended load of 10,000#. As the truss is symmetrically loaded, each reaction is equal to one-half the total vertical load, or 19,000#. To draw the force polygon of the external forces, begin with *ab* and draw *bc*, *cd*, *de*, *ef*, *fg*, *gh* and *hi*. The next force in order is *IJ*, an upward force of 19,000#; therefore measure upward 19,000# locating point *j*. This is followed by the downward forces *jk*, *kl*, *lm*, *mn* and *no*. The remaining force is OA, 19,000#, thus completing the force polygon of the external forces. Beginning with joint $ABPO$ or $HIJZ$, the stress diagram is readily drawn, there being no complications, Fig. 55 (*b*).



(a) Truss Diagram

WARREN TRUSS



(b) Stress Diagram

Fig. 55

Problem 74-A. Construct the stress diagram for the truss and loads shown in Fig. 56 and mark the character and magnitude of the stresses on the truss members.

75. Fan Truss—Suspended Load. A Fan truss having a load of 24,000# on the upper chord and a suspended load of 6,000# is shown in Fig. 57 (*a*). The truss being unsymmetrically loaded, it is necessary to find the magnitudes of the reactions in order to draw the force polygon of the external forces. It is observed that the suspended load IJ has the same line of action as the force CD. Therefore, for the purpose of determining R_1 and R_2, we may assume CD to be 4,000 + 6.000 or 10,000#. Force and funicular

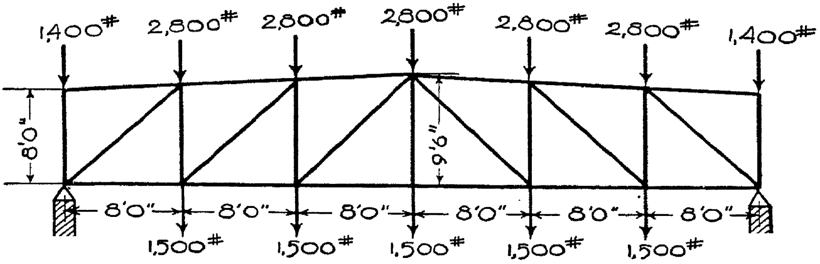

1,400#
2,800#
2,800#
2,800#
2,800#
2,800#
1,400#
8'0"
9'6"
8'0"
8'0"
8'0"
8'0"
8'0"
8'0"
1,500#
1,500#
1,500#
1,500#
1,500#

FIG. 56

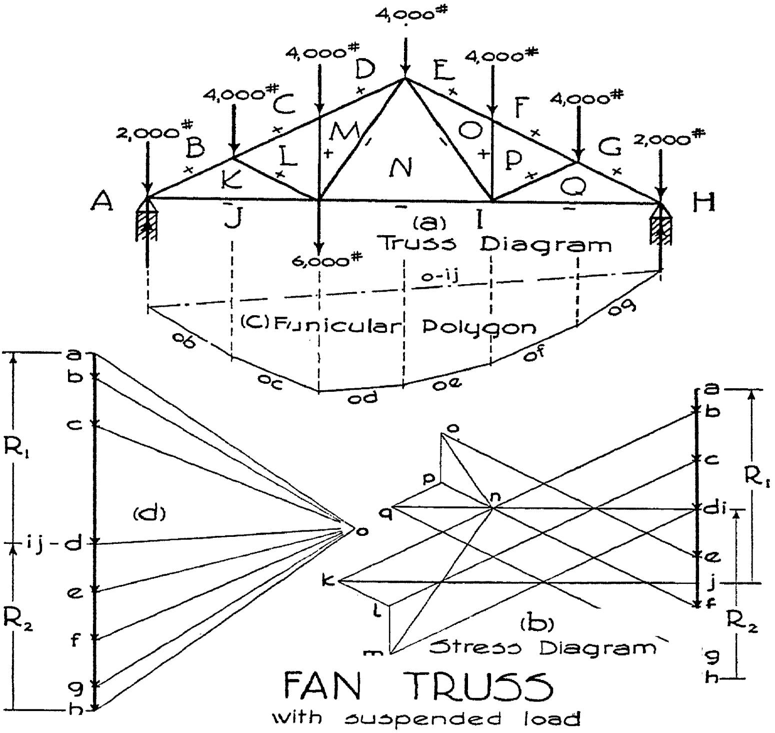

4,000#
4,000#
D
E
4,000#
4,000#
C
F
4,000#
2,000#
B
M
O
P
G
2,000#
L
N
K
Q
A
H
J
I
6,000#
(a) Truss Diagram
o-ij
(C) Funicular Polygon
ob
oc
od
oe
of
og
a
b
c
R₁
(d)
ij-d
e
R₂
f
g
h
o
o
p
n
q
k
l
m
a
b
c
di
e
j
f
g
h
R₁
R₂
(b) Stress Diagram
FAN TRUSS
with suspended load

FIG. 57

polygons are constructed as shown in Fig. 57 (*d*) and (*c*) respectively, and the two reactions thus obtained. This force polygon cannot, of course, be used in drawing the stress diagram since *CD* and *IJ* act at different parts of the truss. Our objective thus far has been merely to find the magnitudes of R_1 and R_2. See Art. 57.

If we had wished, the reactions might have been computed by the principle of moments. The loads are equally spaced horizontally and the resultant of the loads on the upper chord, 24,000#, acts through the center line of the truss. Then

$$6 \times R_1 = (6{,}000 \times 4) + (24{,}000 \times 3)$$

or

$$R_1 = 16{,}000\#$$

and

$$R_2 = (24{,}000 + 6{,}000) - 16{,}000 = 14{,}000\#$$

The polygon of external forces may now be drawn beginning with the downward force *ab* and continuing with *bc*, *cd*, *de*, *ef*, *fg* and *gh*, Fig. 57 (*b*). From point *h* measure upward 14,000#, establishing point *i*, and next lay off *IJ* the downward force of 6,000#. The force *JA*, an upward force of 16,000#, completes the polygon. The stress diagram, as shown in Fig. 57 (*b*), may be started at joint *ABKJ* or *GHIQ*. It presents no difficulties.

PROBLEM 75-A. For the Pratt truss loaded as shown in Fig. 58, draw the stress diagram and mark the character and magnitude of the stresses on the truss members.

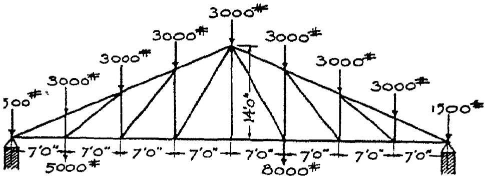


FIG. 58

76. Fink Truss—Substitute Member.

One of the most commonly used steel trusses is the Fink truss shown in Fig. 59 (*a*). The truss diagram shows both vertical and wind loads, and stress diagrams for each will be constructed separately.

Consider first the vertical loads. The truss being symmetrically loaded, the reactions are each equal to one-half the total vertical load. The force polygon of external forces consists then of the loads *ab*, *bc*, *cd*, *de*, *ef*, *fg*, *gh*, *hi* and *ij*, followed by the reactions *jk* and *ka*, Fig. 59 (*b*).

To construct the stress diagram, begin with joint *ABLK* and follow successively with joints *BCML* and *LMNK*. We now find

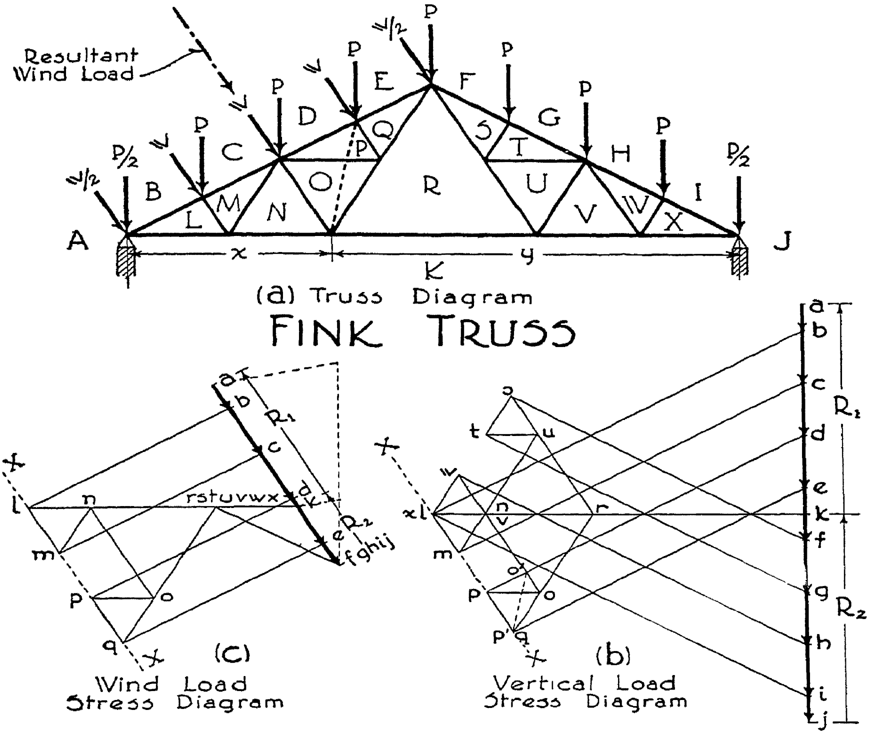


Fig. 59

that joints *CDPONM* and *NORK* each contain more than two unknowns, hence we can proceed no further by the usual methods.

It is noted in the truss diagram that the horizontal member *OP* divides a quadrangle. For the time being, remove *OP* and substitute in its stead the other diagonal shown by the dotted line and call it *O'P'*. This substitution does not disturb the state of equilibrium of the truss. It enables us to draw the stress diagram for the forces about joint *CDO'NM* as there are now only two unknowns.

Therefore, a line through *d* parallel to *DO′*, and a line through *n* parallel to *O′N*, determines point *o′*. As a result of the substitute member, *P′Q* becomes redundant and hence *p′* and *q* will occupy the same point. Hence a line through *e* parallel to member *E—QP′*, and a line through *o′* parallel to *QP′—O′* establishes the point *qp′*. Note particularly that we have now determined the stress in member *EQ*, for the point *q* has been established. The stress in this member has not been affected by the substitute member *P′O′*. We now remove the substitute member *P′O′* and in its place return the original horizontal member *PO*. By drawing a line through point *q* parallel to *QP*, and a line through *d* parallel to *PD* we find point *p*. Next, a line through *p* parallel to *PO*, and a line through *n* parallel to *ON* determines point *o*. By the usual methods point *r* is found, thus completing one-half of the stress diagram. The remaining portion of the diagram is drawn in a similar manner. The method of adding a substitute member is applicable regardless of the magnitudes of the loads, although in the problem just solved the truss is symmetrically loaded. This is the condition most frequently met with in practice.

77. **Fink Truss—Another Solution.** A simpler method of drawing the stress diagram for the Fink truss, *when the truss is symmetrically loaded*, is as follows: Draw the polygon of external forces and complete the stress diagram for joints *ABLK*, *BCML* and *LMNK*. This establishes points *l*, *m* and *n*. Fig. 59 (*b*). For this type of Fink truss, symmetrically loaded and having equal divisions of the upper chord, each successive member of the upper chord, beginning with the member adjacent to the reaction, has the same rate of decrease in stress. By referring to the portion of the stress diagram just completed, note that stresses in *BL* and *CM* have been established and hence the rate of decrease in stress is determined. Members *DP* and *EQ* will have stresses decreased in the same degree. Therefore draw a line through points *l* and *m*, as *XX*. Points *p* and *q* will lie on this line at the intersections, respectively, of lines through *d* parallel to *DP* and through *e* parallel to *EQ*. The remaining portion of the stress diagram is readily completed.

PROBLEM 77-A. For the truss and loads shown in Fig. 60, draw the stress diagram, using the substitute member method as explained in Art. 76, and mark the character and magnitude of the stresses on the truss members.

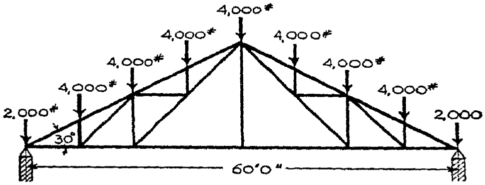


FIG. 60

78. Wind-Load Stress Diagram—Fixed Ends. As well as the vertical loads shown on the Fink truss in Fig. 59 (*a*), loads for wind left are indicated. Assuming the truss to have fixed-ends (no roller), let it be required to draw the stress diagram for the wind loads only. To draw the polygon of external forces, draw the wind loads *ab*, *bc*, *cd*, *de* and *e—fghij*, Fig. 59 (*c*). In drawing a stress diagram it is well to include *all* the letters shown on the truss diagram. In this instance, since there are no loads between letters F, G, H, I and J, these letters will fall at one point. The load line having been drawn, the next two forces to consider are the reactions R_2 and R_1. It is observed in the truss diagram that the wind loads are symmetrical with respect to the left side of the truss, consequently the resultant of the wind loads will act at the midpoint of the upper chord as shown. If the loading had been unsymmetrical, we could have drawn a funicular polygon to find the line of action of the resultant. The extended resultant wind load divides the lower chord into the two segments x and y. Assuming the reactions to be parallel to the direction of the wind, in accordance with the discussion in Art. 59, we know that $R_1 : R_2 :: y : x$. Hence the load line is divided in the same ratio and R_1 and R_2 are determined, establishing point k.

The polygon of external forces having been completed, the stress diagram is drawn using the methods explained in Art. 76 or 77. Attention is called to the fact that letters r, s, t, u, v, w and x occupy the same point in the stress diagram. This indicates that no stresses occur in the web members for the right-hand side of the truss as a result of wind loads on the left. However, these members are stressed when the wind comes from the right, and under this condition zero stresses occur in the web members in the left-hand side of the truss.

PROBLEM 78-A. Assume the total wind load on the left side of the truss in Fig. 60 to be 12,000# and that the reactions are parallel. Draw the stress diagram for wind left and mark the character and magnitude of stresses on the truss members.

79. **Pratt Truss Stress Diagrams.** Fig. 61 (*a*) represents a Pratt truss subjected to both vertical and wind loads. A roller occurs under the right-hand end of the truss.

The stress diagram for the vertical loads is shown at Fig. 61 (*b*). Note that a roller at a support has no effect on the stresses for vertical loading, each reaction being vertical. The stress diagram presents no difficulties.

Let it be required to draw the stress diagram for wind left, a roller being at the right support. To draw the polygon of external forces, first draw the load line *ab*, *bc*, *cd*, *de* and *e—fghij*, Fig. 61 (*c*). Next come the reactions. Assume for the moment that no roller occurs at R_2 and that the reactions are parallel to the direction of the wind. Since the wind loads are symmetrical with respect to the upper chord on the left side of the truss, the resultant wind load is known to act through joint *CDONM* and divides the lower chord into segments *x* and *y*. By the method of proportion explained in Art. 59, the load line is divided into two parts, R_1 and R_2, determining point *k′*. These are the reactions assuming them to be parallel. However, since a roller occurs at R_2, this reaction must be vertical. That is to say, the actual reaction at R_2 can be only the vertical component of the reaction found for the assumption that the reactions are parallel. Hence, erecting a vertical line from point *fghij* and drawing a horizontal line through point *k′*, their intersection determines the true point *k* and consequently the actual vertical reaction at R_2. The reaction at the left side of the truss, *KA*, is found by joining the points *k* and *a*; thus the polygon of external forces is completed. To review the methods of finding R_1 and R_2 see Arts. 60 and 61. The stress diagram is now readily drawn. Observe that, for wind left, no stresses occur in the web members at the right-hand side of the truss since letters *r*, *s*, *t*, *u*, *v*, *w*, *x* and *y* fall at the same point.

For the wind right, roller right, the same methods may be employed. The polygon of external forces consists of the load line *abcde—f*, *fg*, *gh*, *hi* and *ij*, Fig. 61 (*d*). The point *k′* is found as for wind left, based on the assumption that the reactions are parallel.

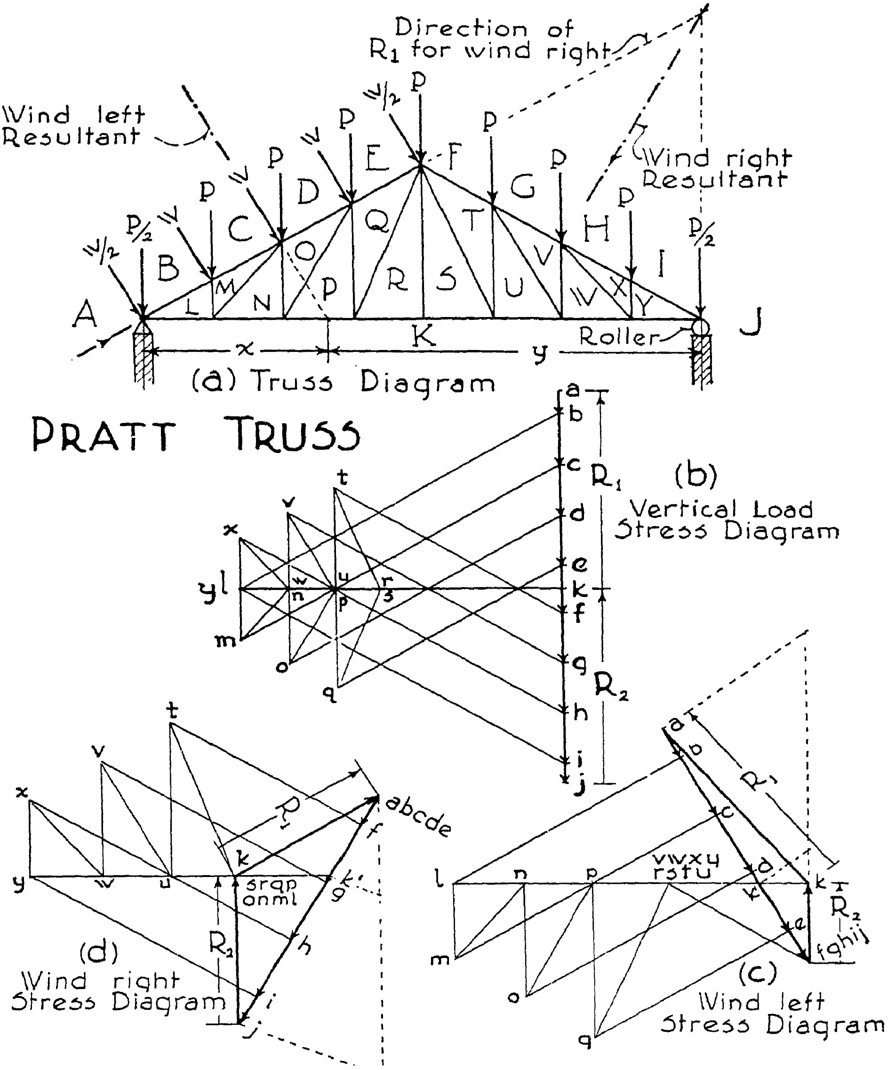

Direction of
R1 for wind right
Wind left
Resultant
Wind right
Resultant
W/2
P/2
W
P
A B C D E F G H I J
K L M N O P Q R S T U V W X Y
Roller
x
y
(a) Truss Diagram
PRATT TRUSS
(b)
Vertical Load
Stress Diagram
R1
R2
(c)
Wind left
Stress Diagram
(d)
Wind right
Stress Diagram
abcde
srqp
onml
vwxy
rstu
fghij

FIG. 61

Since, however, there is a roller at R_2 the reaction cannot be jk' but will be the vertical component of jk' or jk. The stress diagram is completed without difficulty.

The force polygon of external forces might have been drawn by the following simple method. First draw the load line for wind left *abcde—f*, *fg*, *gh*, *hi* and *ij*, Fig. 61 (*d*). Now, since the three external forces, the resultant wind load, R_1 and R_2, are not parallel and are in equilibrium they must have a point in common. Therefore, extend the lines of action of R_2, known to be vertical, and the resultant wind load, until they intersect. A line drawn from this intersection to the point of support at R_1 determines the *direction* of R_1 or KA. Hence in the force polygon, Fig. 61 (*d*), erect a vertical line from *j*. This is the direction of JK and point *k* lies at some point on this line. Again draw a line through point *abcde* parallel to the direction, just found, of KA. Since *k* is on this line as well as on the vertical line through *j*, it is, of course, at their point of intersection.

By inspecting the truss diagram, Fig. 61 (*a*), it is seen that the left reaction for wind right has the same line of action as the left-side upper chord. This fact is verified by the stress diagram and shows that the web members and lower chord on the left side of the truss receive no stresses from the wind loads on the right. The fact that the left reaction and the upper chord are parallel is a mere coincidence and occurs only for a truss of this pitch. To verify this, construct a force polygon for a truss having a ⅓ pitch.

PROBLEM 79-A. The Howe truss shown in Fig. 62 has a span of 64′0″, a rise of 17′0″ and a roller under the right support. The total vertical load is 20,000# and the total loads for wind left and wind right are each 4,800#. Draw separate stress diagrams for wind left, wind right and vertical loads.

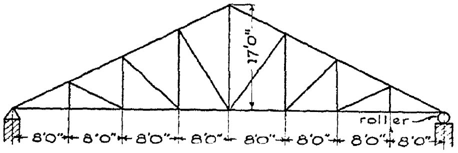
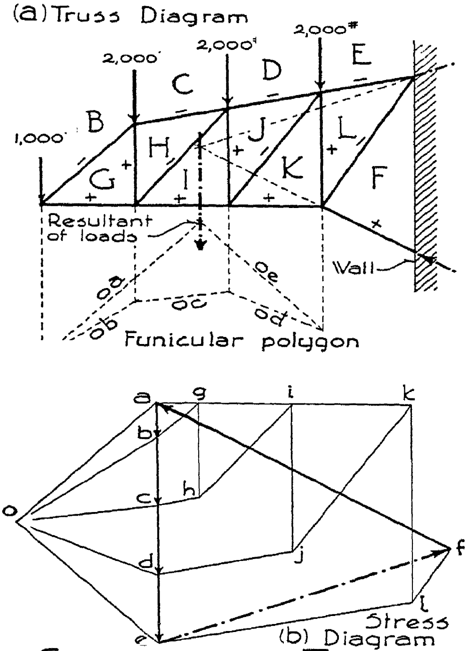


FIG. 62

80. **Cantilever Truss.** A truss that is supported at one end only is known as a cantilever truss. Such trusses are used for roofs over shipping platforms or entrances to buildings. Fig. 63 (*a*) illustrates

a truss of this type. It is presented here not because of its common use, but to illustrate the application of fundamental principles of graphic statics in its analysis.

On inspecting the truss diagram we observe that there are three external forces, the load on the truss, the pull from the wall at joint *LEF* and the compressive force on the wall *FA*. In this instance the truss is not symmetrically loaded and therefore it is necessary to find, by some means, the position of the resultant of the four downward loads. This is accomplished by drawing the load line *ab*, *bc*, *cd*, and *de*, Fig. 63 (*b*), selecting a pole, drawing the rays and finally the funicular polygon. The rays *oa* and *oe* hold the resultant *ae* in equilibrium, hence their intersection in the funicular polygon determines a point through which the resultant must pass.



CANTILEVER TRUSS

FIG. 63

The position of the resultant might readily have been found by mathematics. Although in this truss diagram no dimensions are given showing the position of the loads, it is noted that they are spaced at equal horizontal distances. Then, writing an equation of moments with a point in the face of the wall as an axis,

$$(2{,}000 \times 1) + (2{,}000 \times 2) + (2{,}000 \times 3) + (1{,}000 \times 4) = 7{,}000 \times x$$

or

$$x = 2.28 + \text{spaces}$$

This indicates that the resultant of the loads lies at a point 2.28 spaces from the face of the wall.

The three external forces, being in equilibrium and not being parallel, must have a point in common. The equilibrant at the wall of the force *FA* must have the same line of action as the member *FA*. Therefore, extend the line of action of the member *FA* until it intersects the line of action of the resultant of the loads. A line drawn from this intersection to the joint *LEF* determines the *direction* of the force *EF*.

To draw the force polygon of external forces, from the point *e* in the load line, Fig. 63 (*b*) draw a line parallel to *EF* just found. Next draw a line through point *a* parallel to force *FA*. Their intersection establishes the point *f*, thus completing the force polygon of external forces, Fig. 63 (*b*).

The stress diagram is begun by considering the forces about joint *ABG* and is completed without any complications.

Problem 80-A. For the cantilever truss, shown in Fig. 64, draw the stress diagram and mark the character and magnitude of stresses on the truss members.

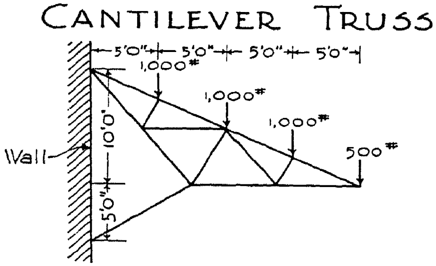
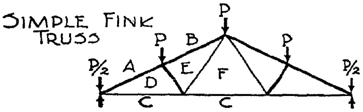


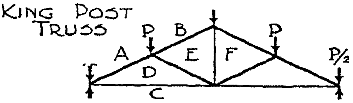
Fig. 64

81. **Stresses Found by Tables.** A knowledge of the principles of graphic statics set forth in this chapter is all that is necessary

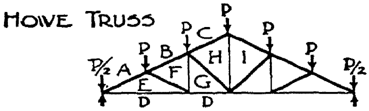
Table IX. Coefficients for Determining Stresses in Simple Trusses



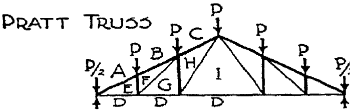
Member	Character of stress	Pitch = height over span ⅓	30°		⅕
AD....	Compression	2.70	3.00	3.35	4.04
BE....	Compression	2.15	2.50	2.91	3.67
DC....	Tension	2.25	2.60	3.00	3.75
FC....	Tension	1.50	1.73	2.00	2.50
DE....	Compression	0.83	0.87	0.90	0.93
EF....	Tension	0.75	0.87	1.00	1.25



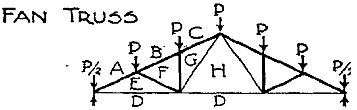
Member	Character of stress	Pitch = height over span ⅓	30°	¼	⅕
AD.	Compression	2.70	3.00	3.35	4.04
BE.	Compression	1.80	2.00	2.24	2.
DC.	Tension	2.25	2.60	3.00	3.75
DE.	Compression	0.90	1.00	1.12	1.35
EF.	Tension	1.00	1.00	1.00	1.00



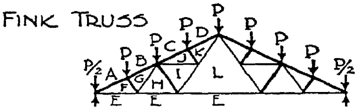
Member	Character of stress	Pitch = height over span	30°		⅕
AE....	Compression	4.51	5.00	5.59	6.73
BF....	Compression	3.61	4.00	4.50	5.39
CH....	Compression	2.70	3.00	3.30	4.04
ED....	Tension	3.75	4.33	5.00	6.2
GD....	Tension	3.00	3.46	4.00	5.0
EF....	Compression	0.90	1.00	1.10	1.3
FG....	Tension	0.50	0.50	0.50	0.50
GH....	Compression	1.25	1.32	1.40	1.6
HI....	Tension	2.00	2.00	2.00	2.0



Member	Character of stress	Pitch = height over span ⅓	30°	¼	⅕
AE.	Compression	4.51	5.00	5.59	6.73
BF.	Compression	4.51	5.00	5.59	6.73
CH.	Compression	3.61	4.00	4.47	5.39
ED.	Tension	3.75	4 33	5.00	6.25
GD.	Tension	3.00	3 46	4.00	5.00
ID.	Tension	2.25	2 60	3.00	3.75
EF.	Compression	1.00	1 00	1.00	1.00
FG.	Tension	1.25	1.32	1.41	1.60
GH.	Compression	1.50	1.50	1.50	1.50
HI.	Tension	1.68	1.73	1.80	1.95



Member	Character of stress	Pitch = height over span ⅓	30°		
AE....	pression	4.51	5.00	5.59	6.7
BF....	pression	3.54	4.00	4.55	5.5
CG....	pression	3.40	4.00	4.71	5.9
ED....		3.75	4.33	5.00	6.2
HD....	Tension	2.25	2.60	3.00	3.7
EF....	pression	0.93	1.00	1.08	1.2
FG....	npression	0.93	1.00	1.08	1.2
GH....	Tension	1.50	1.73	2.00	2.5



Member	Character of stress	Pitch = height over span	30°		
AF..	Compression	6.31	00	7.83	9.
BG..	Compression	5.76	50	7.38	9.
CJ..	Compression	5.20	00	6.93	8.
DK.	Compression	4.63	50	6.48	8.
FE..	Tension	5.25	06	7.00	8.
HE..	Tension	4.50	5.20	6.00	7.
LE..	Tension	3.00	3.46	4.00	5.
FG..	Compression	0.83	0.87	0.89	0.
GH..	Tension	0.75	0.87	1.00	1.
HI..	Compression	1.66	73	1.79	1.
IJ..	Tension	0.75	0 87	1.00	1.
JK..	Compression	0.83	0 87	0.89	0.
KL..	Tension	2.25	60	3.00	3.
IL..	Tension	1.50	73	2.00	2.

for the analysis of the simple trusses met with in everyday practice. Remember that *the first step after finding the loads is to draw the force polygon of external forces.* This having been accomplished, the stress diagram is completed without difficulty. The characters of stresses are found by the method explained in Art. 66 and their magnitudes are obtained by scaling the respective lines in the stress diagram. As has been noted, this graphical solution is one of several methods. It has been adopted in this treatise principally because of its relative simplicity and the fact that the stress diagram is self-checking. To obtain accuracy, stress diagrams should be drawn carefully. Obviously the larger the scale used in drawing the stress diagram, the greater will be the accuracy afforded in determining the magnitudes of the stresses.

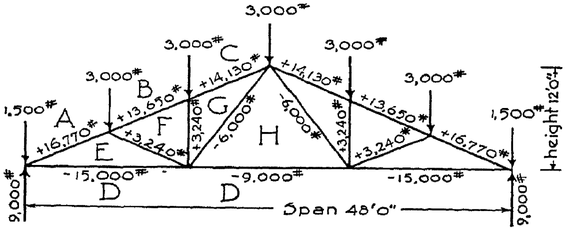


FIG. 65

Table IX, Coefficients for Determining Stresses in Simple Trusses, affords a ready solution of stresses for certain trusses, obviating the construction of stress diagrams. The coefficients given for four commonly used pitches have been determined by mathematics. By use of the table, the stress in any member is obtained by multiplying the panel load P by the stress coefficient found in the column giving the pitch of the truss. Whereas these tables of coefficients may be employed in many instances, their use is limited to trusses symmetrically loaded with vertical loads on the upper chord only. For unsymmetrical loading or for trusses subjected to loads suspended from the lower chord, it is necessary to construct the stress diagram. To show how readily stresses may be found by use of this table, the following illustration is given.

Let it be required to determine the character and magnitude of the stresses in members of a six-panel fan truss having a span of

48′0″ and a height of 12′0″. The total vertical load on the upper chord of the truss is 18,000#, Fig. 65.

A truss having a span of 48′0″ and a height of 12′0″ has a quarter-pitch, corresponding to a slope of 6 in 12, Table I. Since the truss has six equal divisions of the upper chord and the total vertical load is 18,000#, each panel load will be 18,000 ÷ 6, or 3,000#. The half-panel loads at the supports will be 1,500# each, but these loads have no effect on the stresses in the truss members.

Consider first the truss member *AE*, the portion of the upper chord adjacent to the support. Referring to the fan truss, Table IX, under the column "¼ pitch," we find this member to be in compression; the coefficient is 5.59. Therefore, if we multiply the panel load, 3,000#, by this coefficient, 5.59, we have a compressive stress in member *AE* of 16,770#. How simple! The stresses for the remaining members are found in a similar manner and are shown in Table X. Coefficients are given for members in only one-half the truss; the stresses in the other half are, of course, similar.

Table X. Stresses in Six-Panel Fan Truss

Member	Character of stress	Magnitude of stress
AE	Compression	5.59 × 3,000 = 16,770#
BF	Compression	4.55 × 3,000 = 13,650#
CG	Compression	4.71 × 3,000 = 14,130#
ED	Tension	5.00 × 3,000 = 15,000#
HD	Tension	3.00 × 3,000 = 9,000#
EF	Compression	1.08 × 3,000 = 3,240#
FG	Compression	1.08 × 3,000 = 3,240#
GH	Tension	2.00 × 3,000 = 6,000#

PROBLEM 81-A. An eight-panel Fink truss having a span of 63′0″ and a height of 21′0″, has a total vertical load of 25,600# symmetrically distributed at the panel points of the upper chord. By means of Table IX, compute the stresses in the truss members.

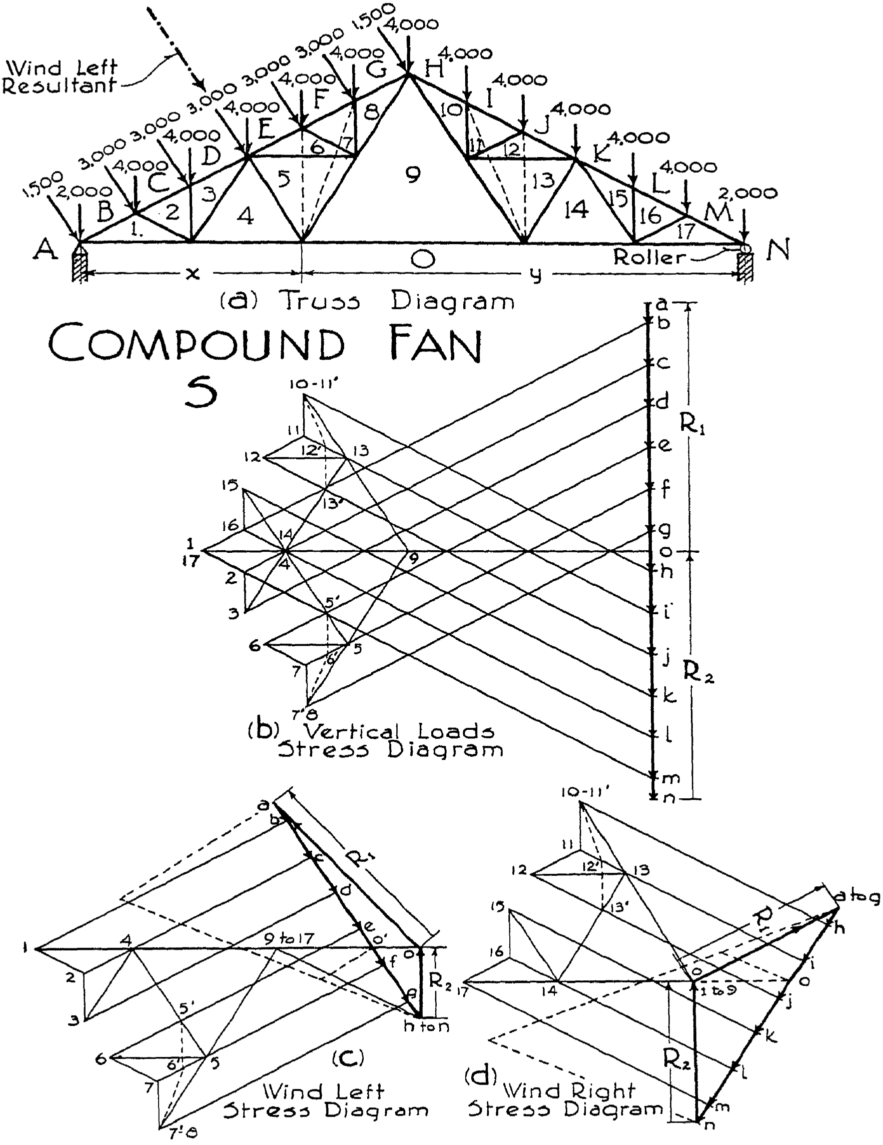
CHAPTER 9

MISCELLANEOUS ROOF TRUSSES

82. **Miscellaneous Roof Trusses.** Chapter 8, immediately preceding, discusses the fundamental principles necessary for the graphical solution of the most commonly used roof trusses. For the average practitioner a thorough knowledge of these principles will enable him to draw the stress diagram for any of the usual trusses. There are, however, certain types of larger trusses that require special attention. Several types of such trusses are discussed in the current chapter, not because they are essential to working knowledge of the subject, but because their solutions exemplify the basic principles formerly discussed and afford a reference for those who wish further investigation.

As has been constantly repeated, after finding the loads, the first step in developing the stress diagram is the construction of the polygon of external forces. In almost every instance, the completion of the stress diagram is comparatively simple after this polygon has been drawn. It will be seen that in practically all of the trusses to be discussed, the construction of the force polygon of external forces is the critical point in the construction of the stress diagram.

83. **Compound Fan Truss.** A twelve-panel compound fan truss having a roller reaction is shown in Fig. 66 (*a*). For the vertical loads, the reactions are vertical and the polygon of external forces is drawn as shown in Fig. 66 (*b*). Beginning with the left support, the joints are taken in the following order, *AB10*, *BC21*, *CD32*, and *12340*. It is found that at joints *DE6543* and *4590* there are more than two unknown forces, consequently we cannot proceed in the usual manner. Instead of the members *56* and *67*, two other members are substituted temporarily. These substitute members are *5'6'* and *6'7'* shown by the dotted lines. We may now proceed with the stress diagram by taking the joints in the following sequence, *DE5'43*, *EF6'5'* and *FG87'6'*, indicating the substitute members as *5'6'* and *6'7'*. Note that when the substitute member *6'7'* is as-


COMPOUND FAN 5

(a) Truss Diagram

(b) Vertical Loads Stress Diagram

(c) Wind Left Stress Diagram

(d) Wind Right Stress Diagram

FIG. 66

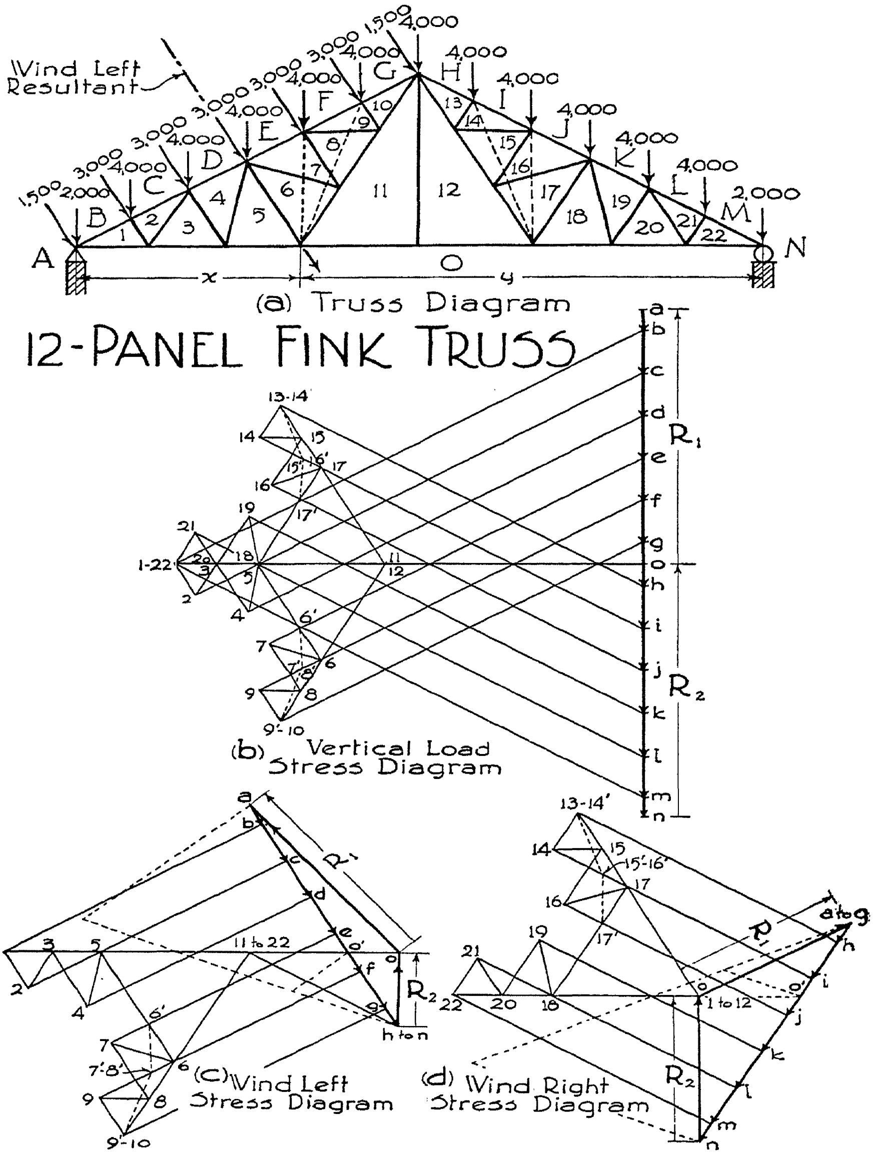

Wind Left Resultant
1,500
2,000
3,000
4,000
A B C D E F G H I J K L M N O
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22
x
y
(a) Truss Diagram
12-PANEL FINK TRUSS
a b c d e f g o h i j k l m n
R1
R2
13-14'
15'-16'
17'
1-22
6'
7'-8'
9'-10
(b) Vertical Load Stress Diagram
11 to 22
h to n
(c) Wind Left Stress Diagram
a to g
1 to 12
(d) Wind Right Stress Diagram

FIG. 67

sumed, a zero stress occurs in member *7′8*, consequently points *7′* and *8* occur at the same position in the stress diagram. In the stress diagram we now see that the stress in *G8* has been determined. This is the actual stress in this member regardless of whether or not the substitute members are introduced. *G8* having been established, we may now discard the substitute members *5′6′* and *6′7′* and replace the original members *56* and *67*. The joints may now be taken in this sequence: *FG87*, *EF76*, *DE6543* and *4590*. This accounts for all the letters and figures in the left side of the truss and the right side is completed in a similar manner, the stress diagram being symmetrical.

The stress diagram for wind left is shown in Fig. 66 (*c*). By the method of the proportional line, explained in Art. 59, the point *o′* is found, based on the assumption that the reactions are parallel. Since, however, there is a roller at R_2, *NO*, the right reaction, can resist only the vertical component of the reaction just found. This vertical reaction is shown in Fig. 66 (*c*), and a line drawn from point *o* to point *a* determines the left reaction, R_1. In the same manner as described for the vertical load stress diagram, the substitute members are introduced and the stress diagram is developed as has been explained.

The stress diagram for wind right is constructed similarly as shown in Fig. 66 (*d*).

84. **Twelve-Panel Fink Truss.** A number of trusses are solved by the method of adding substitute members as described in the previous article. Fig. 67 (*a*) shows a twelve-panel Fink truss. The stress diagrams for vertical loads, wind left and wind right are developed by the same methods employed for the compound fan truss described in Art. 83.

85. **Crescent Truss—Suspended Loads.** A crescent truss, sometimes known as the curved truss, is shown in Fig. 68 (*a*). The total vertical load on the upper chord is 32,000# and the suspended load is 14,000#, making a total vertical load of 46,000#. These loads being uniformly distributed, each reaction is equal to 46,000 ÷ 2 or 23,000#. The force polygon for the vertical loads consists first of the downward forces on the upper chord, *ab*, *bc*, *cd*, *de*, *ef*, *fg*, *gh*, *hi* and *ij*, Fig. 68 (*b*). This is followed by *jk*, an upward force of 23,000#, the right reaction. Next come *kl*, *lm*, *mn*, *no*, *op*, *pq* and *qr*, the suspended loads, and finally the upward force *ra*, 23,000#,

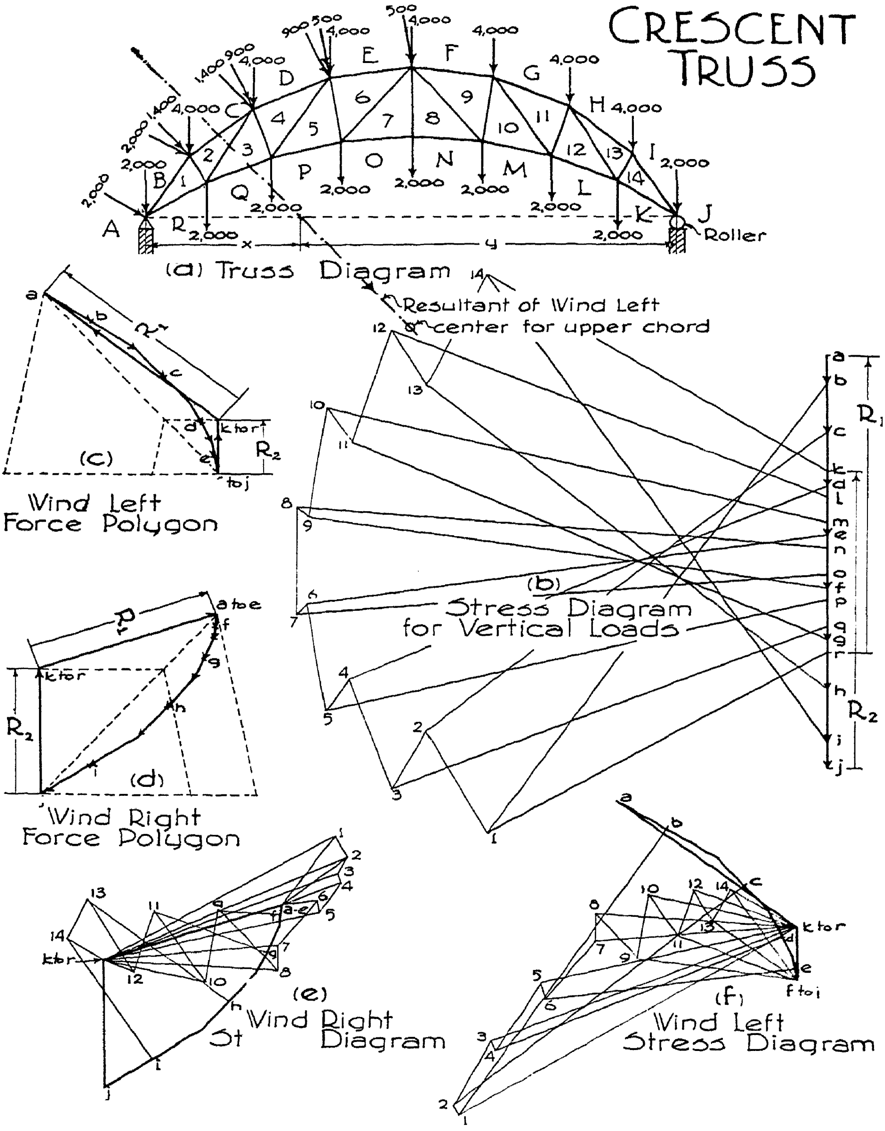

CRESCENT TRUSS
(a) Truss Diagram
A
B
C
D
E
F
G
H
I
J
K
L
M
N
O
P
Q
R
Roller
x
y
2,000
4,000
1,400
900
500
(b)
Stress Diagram for Vertical Loads
Resultant of Wind Left
center for upper chord
R1
R2
(c)
Wind Left Force Polygon
k to r
to j
(d)
Wind Right Force Polygon
a to e
(e)
Wind Right Stress Diagram
a-e
(f)
Wind Left Stress Diagram
f to i

FIG. 68

completes the polygon. The stress diagram may now be completed in the usual manner as shown in Fig. 68 (*b*).

The polygon of external forces for wind left is shown separately in Fig. 68 (*c*). There being four different pitches in the upper chord of the left half of the truss, the magnitudes of the wind loads on the four roof surfaces vary as well as their directions. The four forces shown are 4,000#, 2,800#, 1,800# and 1,000#. One-half of each of these forces is transferred to their adjacent panel points. Our first objective is to find the resultant of these different wind loads. Of course we might draw a funicular polygon to find their resultant, but perhaps a simpler method is as follows. Beginning with the wind load *AB*, 2,000#, start the force polygon. Between spaces *B* and *C* there are two wind loads, 2,000# and 1,400#. These two forces are laid off as shown in Fig. 68 (*c*). From space *C* to *D* there are the two forces, 1,400# and 900#; from *D* to *E* we have forces 900# and 500#; and from *E* to *F* there is the remaining wind load of 500#. There being no wind forces on the right side of the truss, the letters *f*, *g*, *h*, *i* and *j* fall at one point. The load line for wind left therefore consists of *ab*, *bc*, *cd*, *de* and *e-f* to *j*, and the resultant of the wind loads is the dotted line in the force polygon drawn from point *a* to point *fghij*, Fig. 68 (*c*). Thus we have determined the magnitude and direction of the resultant.

Referring to the truss diagram, Fig. 68 (*a*), we find that the upper chord of the truss has the shape of an arc of a circle and that the center of the circle is designated. Since the wind loads on the different parts of the upper chord are normal to the chord, each separate wind load will pass through this center of the circle. It is a fundamental principle that the resultant of a number of concurrent forces passes through their point of intersection. Therefore, through the center of the circle draw a line parallel to the resultant wind load found in the force polygon, Fig. 68 (*c*). We have now determined the resultant in magnitude, direction and line of action. In drawing the polygon of external forces for wind loads, note that letters *k*, *l*, *m*, *n*, *o*, *p*, *q* and *r* fall at the same point.

By the method described in Art. 59, R_1 and R_2 are found assuming the reactions to be parallel. There being a roller at R_2, this reaction must be vertical and the true reactions are consequently found as indicated and identified in Fig. 68 (*c*). This completes the force polygon of external forces for wind left, and the stress diagram is drawn in the usual manner as shown in Fig. 68 (*f*).

The polygon of external forces for wind right is constructed in a similar manner, as shown in Fig. 68 (*d*), the stress diagram being shown in Fig. 68 (*e*).

86. **Combined Vertical and Wind Loads.** A six-panel crescent truss having vertical and wind loads with a roller at the right support is shown in Fig. 69 (*a*). The magnitude of the stresses may be found by constructing separate stress diagrams as explained in Art. 85. A method sometimes employed, however, is to draw a stress diagram for wind left and vertical loads combined, also a combined stress diagram for wind right and vertical loads.

Fig. 69 (*d*) shows the polygon of external forces for wind left and vertical loads combined. Referring to the truss diagram, from *A* to *B* there are two forces, a vertical load of 2,000# and a wind load of 2,500#. Therefore, beginning with point *a*, Fig. 69 (*d*), draw a downward force of 2,000# and a force of 2,500# downward to the right. This determines point *b*. From *B* to *C* there are three forces, 2,500#, 4,000# and 1,500#, having the directions shown. From point *b* draw these three forces, thus determining point *c*. In like manner draw from point *c* the three forces 1,500#, 4,000# and 500#, establishing point *d*, and from *d* draw the forces 500# and 4,000#, determining point *e*. From point *e* we draw *ef*, *fg* and *gh*, 4,000#, 4,000# and 2,000# respectively. Thus is drawn the load line, and the resultant of wind left and vertical loads combined is represented in magnitude and direction by the dotted line from *a* to *h*.

The center of the circle, from which the arc of the upper chord is drawn, is indicated in Fig. 69 (*a*). Since, by construction, the lines of action of the wind loads all pass through this point as well as the resultant of the vertical loads, the line of action of the resultant of the combined wind and vertical loads must pass through this center point. Therefore, through the cênter point draw a line parallel to the resultant *ah* found in Fig. 69 (*d*). This represents the line of action of the resultant of all the loads and divides the line connecting the two reactions into segments x and y. As has been explained in previous problems, R_1 and R_2, the two reactions are now found, R_2 being vertical since there is a roller at this support. A separate diagram showing the complete stress diagram for wind left and vertical loads combined is shown in Fig. 69 (*b*).

Fig. 69 (*e*) is the polygon of external forces for wind right and vertical loads combined, and the complete stress diagram is shown

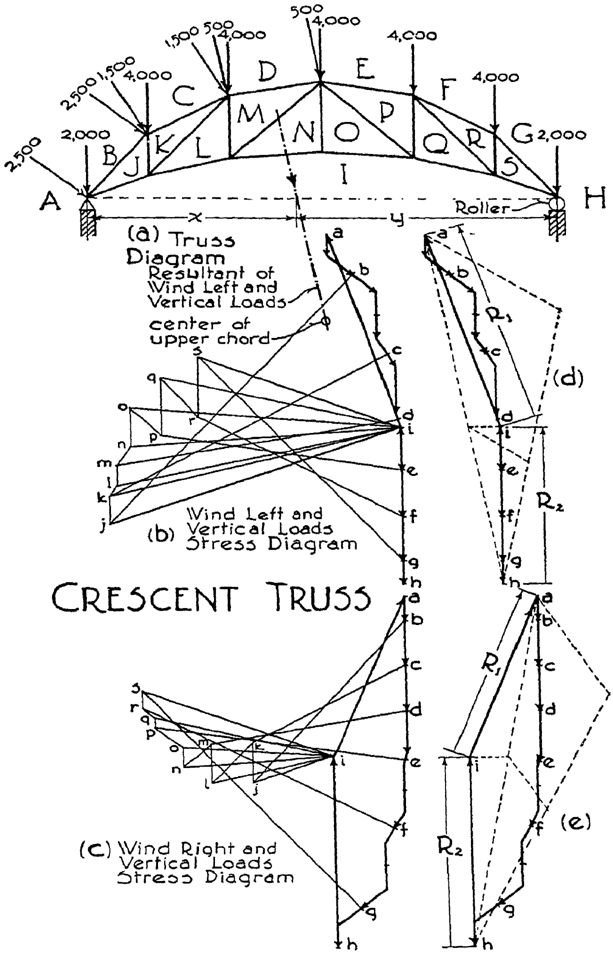


FIG. 69

in Fig. 69 (*c*). Fig. 69 (*e*) is constructed by the same method employed in drawing Fig. 69 (*d*), as previously explained.

87. **Truss for Gambrel Roof.** A truss having two different pitches on each half of the truss and having loads suspended from the lower chord is shown in Fig. 70 (*a*). Let it be required to draw the various stress diagrams for this truss and loading, *assuming the horizontal components of the reactions due to the wind loads to be equal.*

A stress diagram for vertical loads is first drawn, Fig. 70 (*b*). The total vertical load is 46,000# and since the truss is symmetrically loaded, each reaction, *JK* and *PA*, has a magnitude of 23,000#. Beginning with point *a*, the loads on the upper chord are drawn, *ab*, *bc*, etc., to point *j*. Next comes the upward force *jk*, 23,000#, followed by *kl*, *lm*, *mn*, *no* and *op*, the suspended loads. The upward force *pa*, 23,000#, completes the polygon of external forces. In conjunction with this polygon just drawn, the stress diagram is developed, no difficulties being met with, Fig. 70 (*b*).

A separate force polygon for external forces for wind left is shown in Fig. 70 (*e*). By inspecting the truss diagram we note that there are two wind loads for wind left, their magnitudes being 12,000# and 4,000#, each having different directions. These loads are apportioned to the adjacent panel joints. Beginning with point *a*, Fig. 70 (*e*), draw the forces *ab*, *bc*, *cd*, *de* and *ef*, the complete load line for wind left. In drawing this load line note that there are two forces between letters *C* and *D* in the truss diagram, so that actually the force *CD* is the resultant of the forces 3,000# and 1,000#. To account for all the letters shown on the truss, note that for wind left the letters *f*, *g*, *h*, *i* and *j* fall at one point, and likewise the letters *k*, *l*, *m*, *n*, *o* and *p* occupy the same position. The resultant of the wind loads in magnitude and direction is the line drawn from point *a* to point *fghij*. Since the resultant of two concurrent forces passes through their point of intersection, the line of action of the resultant of the wind left forces will pass through point *x*, see Fig. 70 (*a*). Assuming, temporarily, the reactions to be parallel, R_1 and R_2 are found by the proportional line method as indicated in Fig. 70 (*e*). Data for this problem, however, calls for the horizontal components of the reactions to be equal, therefore R_1 and R_2 are determined as explained in Art. 59. Referring to Fig. 70 (*e*), note that H_1 and H_2 are the equal horizontal components of R_1 and R_2 respectively. Using the force polygon just completed, a separate stress diagram for wind left is developed in Fig. 70 (*c*).

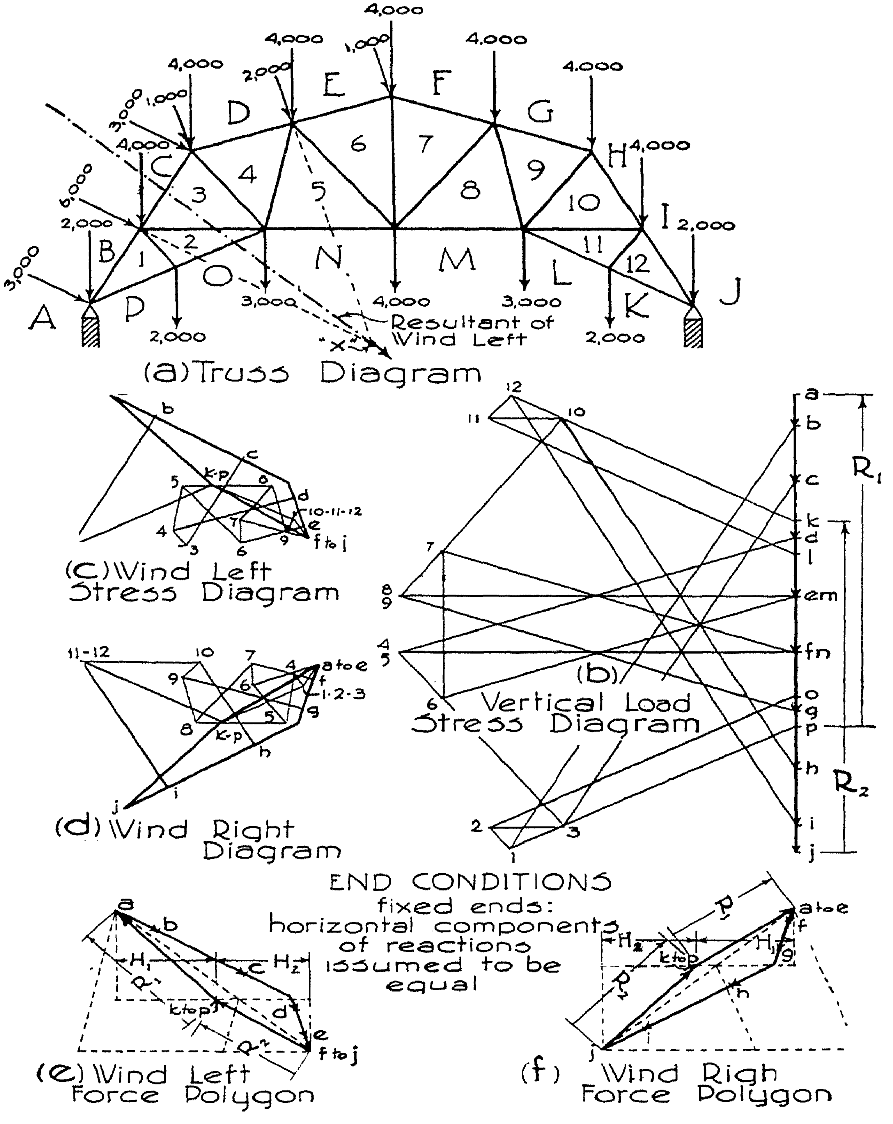
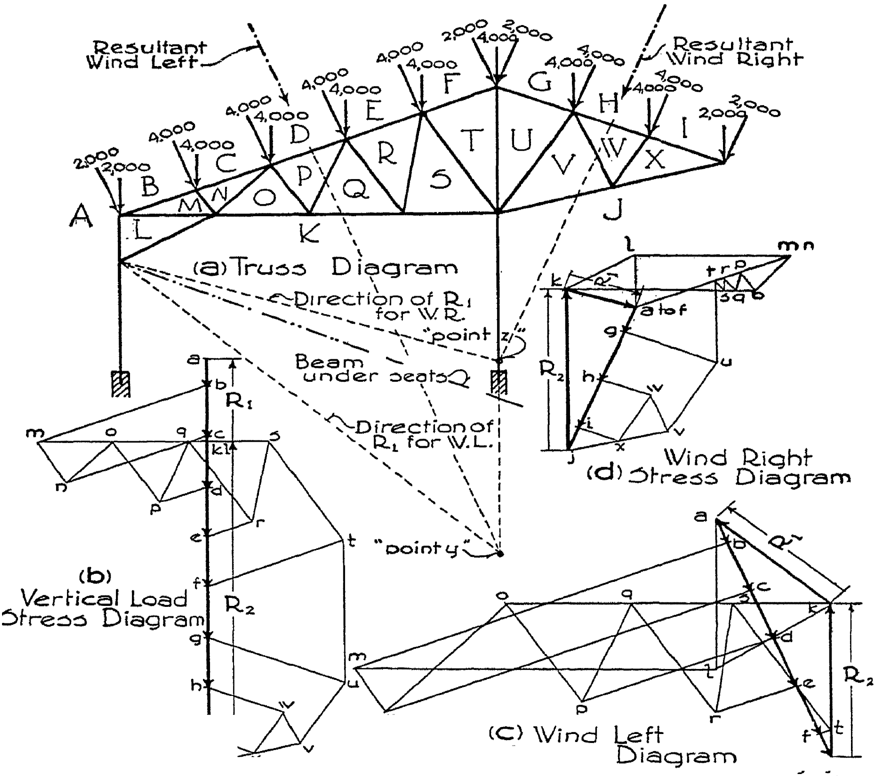

(a) Truss Diagram
4,000
4,000
4,000
4,000
4,000
4,000
4,000
2,000
2,000
3,000
1,000
2,000
1,000
6,000
3,000
2,000
3,000
4,000
3,000
2,000
A B C D E F G H I J K L M N O P
1 2 3 4 5 6 7 8 9 10 11 12
Resultant of Wind Left
"x"
(b) Vertical Load Stress Diagram
a b c k d l em fn o g p h i j
R1
R2
(c) Wind Left Stress Diagram
k-p
10-11-12
f to j
(d) Wind Right Diagram
11-12
a to e
1-2-3
k-p
END CONDITIONS
fixed ends:
horizontal components of reactions assumed to be equal
(e) Wind Left Force Polygon
H1
H2
R1
R2
k to p
f to j
(f) Wind Righ Force Polygon
a to e
H2
H1
R1
R2
k to p

Fig. 70

In exactly the same manner, the force polygon of external forces for wind right and the complete stress diagram are constructed as shown in Figs. 70 (*f*) and (*d*).

88. **Grandstand Truss—Wind Loads.** A type of truss used for grandstands, having both vertical and wind loads is shown in Fig. 71 (*a*).



GRANDSTAND TRUSS

FIG. 71

To construct the polygon of external forces for the vertical loads, the load line is first laid off, *ab*, *bc*, *cd*, etc., ending with *ij*, the total load being 32,000#, Fig. 71 (*b*). The reactions due to these loads are, of course, vertical and may be found by means of the funicular polygon, the method employed in Art. 73 and shown in Fig. 53, or by the principle of moments. Although no dimensions are given on the truss diagram, we observe that the vertical loads occur at equal horizontal distances. Since the loading is symmetrical, the resultant of the vertical loads, 32,000#, has its line of action at the center of the loading, four spaces from the left support or four spaces from the right end of the truss. Writing an equation of moments about the left support as an axis, we have

$$4 \times 32{,}000 = 5 \times R_2$$

or

$$R_2 = 25{,}600\#$$

Since

$$R_1 + R_2 = 32{,}000\#,\ R_1 + 25{,}600 = 32{,}000\#$$

or

$$R_1 = 6{,}400\#$$

From point j in the load line measure upward 25,600#, thus determining point k, and from k to point a we have 6,400#, thus completing the polygon of external forces. The stress diagram is completed without difficulty as shown in Fig. 71 (*b*). Attention is called to the fact that no stress occurs in member KL due to vertical loads, consequently the letters k and l fall at the same point in the stress diagram. It will be seen, however, that this member is stressed when wind loads occur.

In a truss of this type, the column JK is able to resist only vertical forces; hence the horizontal components of the wind loads are resisted by KA, the left reaction. In Fig. 71 (*a*) it is seen that framing for the seats joins the left support at joint ALK. These beams or girders resist the horizontal thrust of either wind right or wind left. Consequently, in constructing the polygon of external forces for wind loads, we must assume that JK is vertical; the direction of KA remains to be determined.

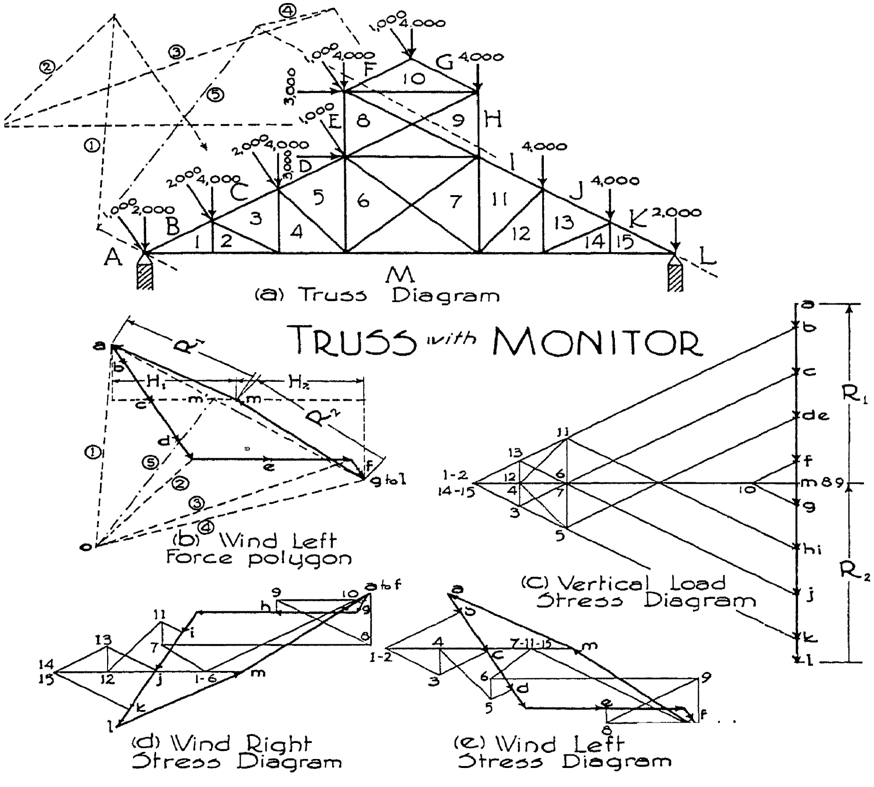
Let us first consider wind left. Begin the polygon of external forces by drawing the load line *ab*, *bc*, *cd*, *de*, *ef* and *f-ghij*, Fig. 71 (*c*). Note that the letters g, h, i and j fall at the same point for wind left. The three external forces are the resultant wind load, R_1 and R_2. These forces, being in equilibrium and not parallel, must meet in a point. As the wind loads are symmetrically placed on the left upper chord, it is unnecessary to draw a funicular polygon to find the position of their resultant. We know its line of action is located at the center line of the upper chord. This line of action is extended to intersect the line of action of R_2 which we know to be vertical, the point of intersection being point *y*. A line drawn from this point to joint *ALK* determines the direction of R_1, the left reaction, for wind left. Returning to the load line, Fig. 71 (*c*), draw a vertical line, the direction of R_2, through point *ghij* and also a line through point *a* parallel to the direction of R_1 just found. Their intersection establishes point *k* and consequently R_1 and R_2 are established and the polygon of external forces is completed. The development of the stress diagram in conjunction with this polygon presents no difficulties. There being no wind loads on the right side of the truss, no stresses occur in members to the right of the right support and therefore the letters *u, v, w* and *x* occur at the same point as *g, h, i* and *j*.

The polygon of external forces for the wind right stress diagram is drawn in exactly the same manner. The resultant of the wind loads is extended to intersect the line of action of R_2, at point *z*, and a line from this point to joint *ALK* establishes the direction of R_1 for wind right, Fig. 71 (*d*). In this diagram observe that *k—abcdef*, which is R_1, reads downward to the right, indicating that the beam under the seats is in tension due to wind coming from the right.

89. **Truss with Monitor.** There are several different arrangements of members for the truss with a monitor shown in Fig. 72 (*a*). In the central portion of the truss shown, there are two quadrilateral spaces, each of which is divided by two diagonals. It will be seen on drawing the stress diagrams that these diagonal members are not subjected to stresses when the loading is vertical and uniformly distributed. When, however, wind loads occur, these members are stressed and the diagonal members to be considered are those which are in tension.

The polygon of external forces for vertical loads is readily drawn. *LM* and *MA*, the two reactions, are vertical and each is equal in magnitude to 16,000#, Fig. 72 (*c*). The stress diagram is drawn in the usual manner, it being observed that for symmetrical vertical loads no stresses occur in the diagonals 89 or 67.

Let us now draw the polygon of external forces for wind left, assuming the horizontal components of the reactions to be equal. Beginning with point *a* draw *ab*, *bc*, *cd*, *de*, *ef* and *fg*, Fig. 72 (*b*). Observe that both *DE* and *EF* are actually the resultant of two wind loads having different directions. Having completed the load line, first assume the reactions *LM* and *MA* to be parallel to the direction of the resultant of the wind loads. The resultant of the wind loads is the line drawn from point *a* to point *ghijkl*. Therefore, draw the dotted lines through joints *KLM 15* and *AB1M*, Fig. 72



(a) Truss Diagram

(b) Wind Left Force polygon

(c) Vertical Load Stress Diagram

(d) Wind Right Stress Diagram

(e) Wind Left Stress Diagram

FIG. 72

(*a*), parallel to this resultant. These lines represent the lines of action of the reactions based on the assumption that they are parallel to the direction of the wind-load resultant. To find the magnitude of the reactions with this assumption, we construct the funicular polygon. The pole *o* is selected, Fig. 72 (*b*), the rays drawn and the funicular polygon constructed as shown in Fig. 72 (*a*). In constructing the funicular polygon there are five forces in equilibrium, the wind load on the lower upper chord, the wind load on the vertical side of the monitor, the wind load on the upper part of the monitor and the two reactions. The closing string of the funicular polygon is the dot-and-dash line number 5. Then if we draw, parallel to the closing string, a line from the pole *o* to the resultant of the wind loads we establish point *m'*, thus determining the reactions based on the assumption that they are parallel. By data, however, we are to consider the horizontal components of the reactions to be equal; hence, using the method explained in Art. 59 and shown in Fig. 38, the point *m* is determined and consequently the two reactions. This completes the polygon of external forces.

To avoid a confusion of lines, this polygon of external forces is redrawn in Fig. 72 (*e*) to be used in constructing the stress diagram. In the two quadrangular spaces in the central portion of the truss two diagonals are shown, whereas we know that one diagonal in each space is all that is necessary. For the wind left we will consider only the diagonals that receive tensile stresses. This can readily be determined by trial. First let us draw the stress diagram, assuming the diagonals 89 and 67 to be those shown having directions upward to the right, assuming the remaining pair to be omitted. The stress diagram is now drawn taking the joints in the following sequence, *AB1M*, *12M*, *BC321*, *234M*, *CD543*, *FG10*, *EF108*, *GH9810*, *DE8965*, etc. By using the method of determining the character of stress explained in Art. 66, we find that the two diagonals 89 and 67 are each in tension; hence our assumption was correct.

The stress diagram for wind right is shown in Fig. 72 (*d*). For wind right, the two diagonals whose directions are upward to the left are considered and these are found to be in tension. The use of two sets of diagonal members is for economy. If only one set were used they would be in tension for wind in one direction and compression for wind in the opposite direction, a reversal of stress. Since, for comparatively long members, less material is required for members in tension, both members are employed, each being designed to resist tension.

90. **Transverse Bent.** For certain commercial buildings a common type of construction is a series of trusses supported on columns, the trusses being braced to the columns by sloping members

called *knee-braces*. The name given to such a unit is a *transverse bent* as illustrated in Fig. 73 (*a*).

The truss shown is an eight-panel Fink truss, the knee braces being members *A3* and *A 17*. For vertical loads, the reactions are

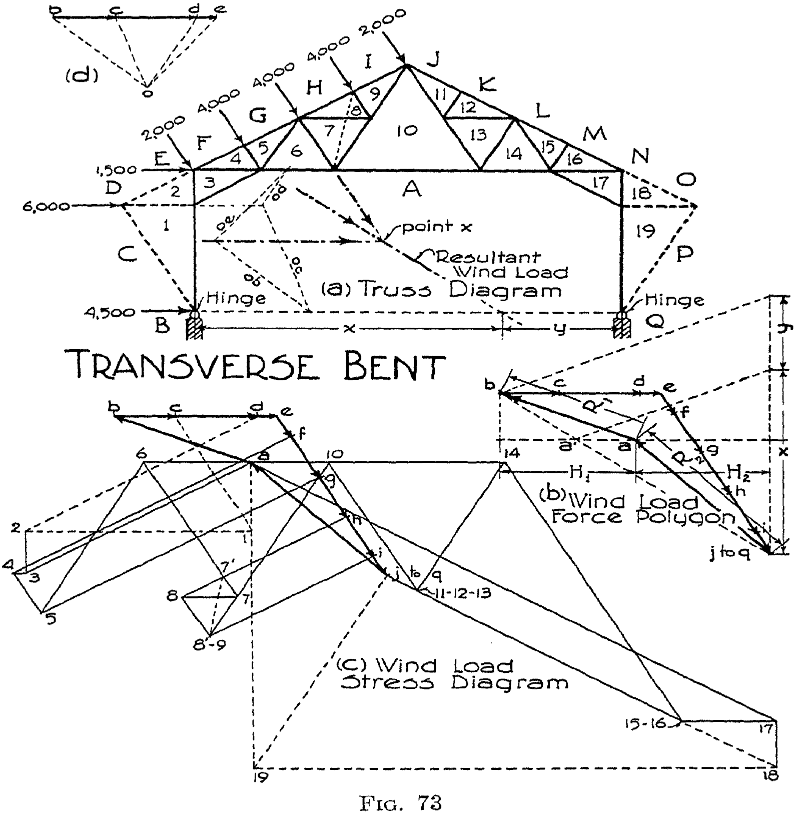


FIG. 73

equal and vertical and no stresses occur in the knee-braces, the stress diagram being constructed as described in Arts. 76 and 77 and shown in Fig. 59.

The usual construction is to secure the base of the column to the masonry foundation by means of anchor bolts. Such anchorage is not sufficiently rigid to produce a "fixed end," hence in designing the column it is assumed that no bending moment occurs at the base of the column. Therefore, for this form of connection, we consider the column to be hinged at the base. If, however, the column is deeply embedded in the masonry so as to produce a bending moment at this point when the truss is subjected to wind loads, it is safe to assume the inflection point in the column to occur at half the distance from the base to the knee-brace. For this condition we may assume the column to be hinged at the inflection point and ignore the wind loads acting below this point. Since, for practical considerations, it is quite difficult to construct a column with a base that is actually "fixed," it is generally assumed that the column is hinged at the base and this condition will be considered in the following solution.

In designing the columns for bending due to wind loads, the maximum bending moment occurs in the column on the leeward side of the structure at the point at which the knee-brace joins the column, its magnitude being the horizontal component of the reaction multiplied by the distance from the knee-brace to the base of the column.

The truss diagram shown in Fig. 73 (*a*) is actually an incomplete frame if we assume the structure to be hinged at the joints. For this theoretical condition the frame is unstable. Therefore, to enable us to construct a stress diagram for the truss, we assume that the imaginary framing, shown by the dotted lines on the sides of the columns, is substituted. These members are *C1*, *D2*, and *12* on the left side of the truss and the corresponding members on the right side. The stresses in the columns, as found by the stress diagram, are not true stresses, for by construction we know the columns to be in bending. However, the stress diagram gives the true stresses in all the members of the truss including the knee-braces. In the stress diagram the substitute framing members are shown by dotted lines and, of course, are ignored in designing the truss.

In the design of a building of this type, the wind load on the vertical sides of the structure must be considered as well as that on the roof. Referring to the truss diagram, Fig. 73 (*a*), the wind load acting horizontally on the side of the building is 12,000# and the wind load on the roof is 16,000#. These loads are, of course, the loads to be resisted by each truss and are distributed to the panel joints as shown.

The line of action of the resultant of the wind load on the side of the building is found by the force polygon, Fig. 73 (*d*), and its corresponding funicular polygon. The resultant wind load on the roof surface acts at the midpoint of the upper chord, having the same line of action as force *GH*. The intersection of these two resultants, point x, determines a point through which the resultant of all the wind loads must pass. To find the magnitude and direction of the resultant of all the wind loads, beginning with point b, draw the load line *bc*, *cd*, *de*, *ef*, *fg*, *gh*, *hi* and *i-jklmnopq*, Fig. 73 (*b*). The resultant of the wind loads, therefore, is the line drawn from point b to point *jklmnopq*. A line parallel to this resultant is drawn through point x and divides the line drawn from the bases of the columns into the segments x and y. By first assuming the reactions to be parallel, the point a' is found as shown. Assuming the horizontal components of the reactions to be equal, as explained in Art. 59, the point a is found, thus determining R_1 and R_2.

By use of the polygon of external forces just completed, a separate stress diagram for wind left is drawn as shown in Fig. 73 (*c*). The substitute member 7′8′ is employed as explained in Art. 76. The auxiliary members shown in the truss diagram have corresponding dotted lines in the stress diagram. Since we know the columns to be in bending, the stresses indicated in the stress diagram for the members *A 1* and *A 19* are, of course, not the true stresses.

91. **Three-Hinged Arch—Vertical Loads.** The three-hinged arch is used for long spans such as required by armories and auditoriums. One of the principal advantages in its use is that the crown is permitted, due to the hinges, to rise and fall with temperature variations. In the solution of problems relating to the three-hinged arch the critical problem is the determination of the reactions. Regardless of the loads on the arch, the reactions are not vertical. The horizontal components of the thrusts may be resisted by the masonry foundations or by a tie member connecting the two segments of the truss at the base hinges.

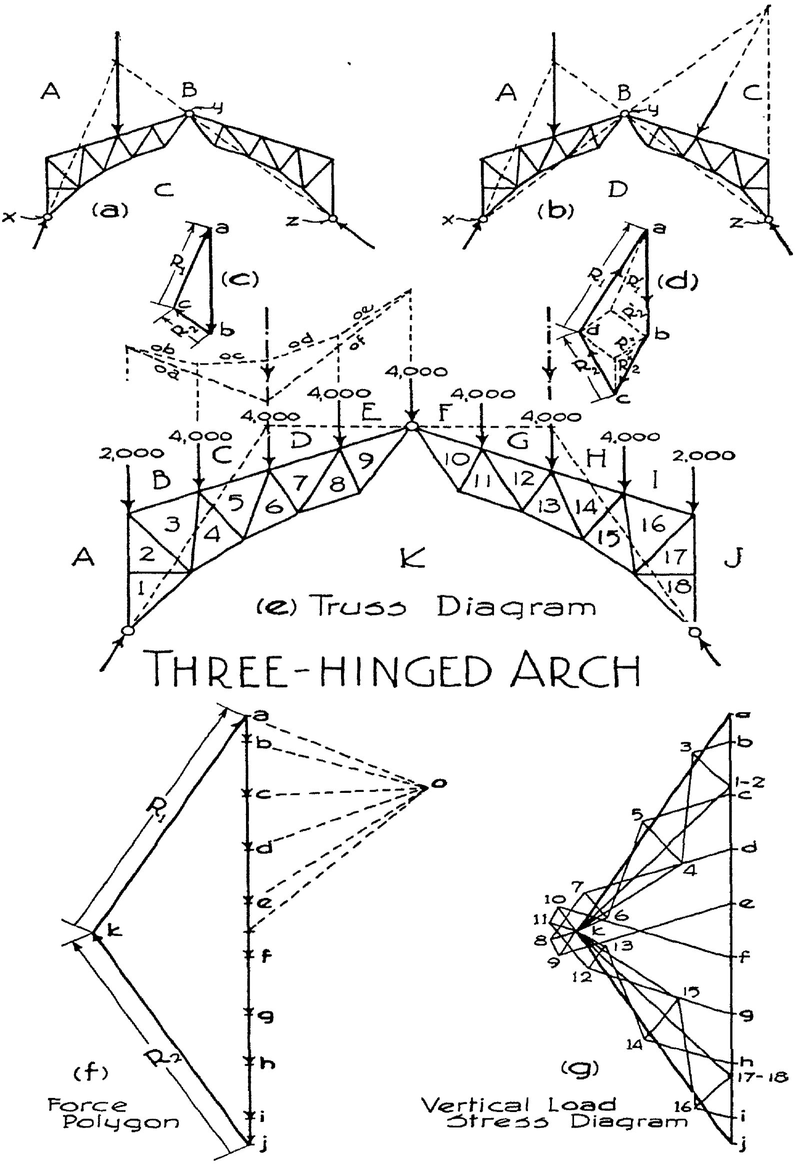
Basically, the three-hinged arch is composed of two separate trusses hinged at the crown and at their points of support. The forces transmitted from one segment of the arch to the other, at the intermediate hinge, must be equal and opposite in direction to comply with the laws of equilibrium.

A three-hinged arch, having the load *AB* on one segment only, is shown in Fig. 74 (*a*). The three hinges are indicated at points *x*, *y* and *z*. To find the reactions *BC* and *CA* first draw the load line *ab*, Fig. 74 (*c*). The forces acting on the left segment are *CA*, *AB* and the force transferred from the right segment at hinge *y*. The right segment is acted upon by two forces, the reaction at hinge *z* and the force transferred from the left segment at hinge *y*. Since there are but two forces acting on the right segment, to produce equilibrium these two forces must be equal, opposite in direction and have the same line of action. Therefore, a line drawn through hinge *z* and hinge *y* determines the direction of the two forces acting on the right segment and consequently the direction of the reaction *BC*. There are three external forces acting on the truss, the load *AB* and the two reactions *BC* and *CA*. Since they are in equilibrium and are not parallel they must meet in a common point. Extend the line of action of the right reaction *BC* until it intersects the force *AB*. From this point of intersection draw a line to the left support, hinge *x*, thus determining the *direction* of *CA*, the left reaction. In the force polygon, Fig. 74 (*c*), draw from point *b* a line parallel to the reaction *BC*, and from point *a* draw a line parallel to the reaction *CA*. Their intersection establishes the point *c* and consequently R_1 and R_2, the two reactions, are now established.

A three-hinged arch with two unequal loads, *AB* and *BC*, on the left and right segments are shown in Fig. 74 (*b*). Let it be required to find the reactions *CD* and *DA*.

This is readily accomplished by considering the loads *AB* and *BC* separately. The load line *ab* and *bc* is first drawn, Fig. 74 (*d*). By the method employed in the previous example, the reactions resulting from the load *AB*, R_1' and R_2' are found. In the same manner the reactions resulting from the load *BC* are found. They are shown in Fig. 74 (*d*) as R_1'' and R_2''. At the left support we have then the reaction due to the load *AB* and also the reaction due to the load *BC*, namely R_1' and R_1''. The resultant reaction from these two reactions is found by the triangle of forces and is the true reaction, R_1, due to the two loads *AB* and *BC*. Similarly the reaction at the right support, R_2, is determined as shown in Fig. 74 (*d*). The force polygon of external forces then is *ab*, *bc*, *cd* and *da*.

A three-hinged arch with vertical loads on the upper chord is shown in the truss diagram, Fig. 74 (*e*). To draw the stress diagram


A
B
y
C
(a)
x
z
A
B
y
C
D
(b)
x
z
a
R_1
c
R_2
b
(c)
a
R_1
R_1'
(d)
R_2'
d
b
R_1'
R_2
R_2'
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J
K
1
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3
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9
10
11
12
13
14
15
16
17
18
(e) Truss Diagram
THREE-HINGED ARCH
a
b
c
d
e
f
g
h
i
j
o
k
R_1
R_2
(f)
Force Polygon
a
b
1-2
c
d
e
f
g
h
17-18
i
j
3
4
5
6
7
8
9
10
11
12
13
14
15
16
k
(g)
Vertical Load Stress Diagram

FIG. 74

we first draw the polygon of external forces. To accomplish this, first draw the load line *ab, bc, cd, . . . ij*, Fig. 74 (*f*). As yet the directions of the reactions *JK* and *KA* are unknown. The forces acting on the left segment of the truss are three in number, the left reaction, the load on the left segment and the force exerted by the right segment at the hinge at the crown. The resultant of the loads on the left segment is found by the funicular polygon as shown, although this step is unecessary in this instance since the loads are symmetrically placed and we know the resultant to have the same line of action as the force *CD*. At the hinge at joint *EF 10 K 9*, each segment exerts a force which holds the opposite segment in equilibrium. These two forces must be equal, opposite in direction and have the same line of action through the hinge. Since the loads on each segment of the truss are equal, the forces transferred from each segment of the truss must be equal and horizontal. Therefore, a horizontal line drawn through the hinge determines the line of action of these two forces in equilibrium. Considering again the three forces on the left segment, the reaction, the load on the left segment and the force exerted by the right segment, these three forces must meet in a common point since they are in equilibrium and are not parallel. Therefore, extend the horizontal line through the hinge at joint *EF 10 K 9* until it meets the resultant of the loads on the left segment. From this point of intersection draw a line to the hinge at the left support, thus determining the direction of the left reaction, R_1. In the same manner determine the direction of the right reaction R_2. From point *j* in the load line, Fig. 74 (*f*), draw a line parallel to the right reaction *JK*, and from point *a* draw a line parallel to the left reaction *KA*. Their point of intersection establishes the point *k* and thus the two reactions, *JK* and *KA*, are determined.

Using the polygon of external forces just found, a separate diagram, Fig. 74 (*g*), is drawn showing the complete stress diagram for vertical loads. As has been seen, the principal problem in the solution of a three-hinged arch is the determination of the reactions. After this has been accomplished the stress diagram presents no difficulties.

92. **Three-Hinged Arch—Wind Loads.** Let it be required to draw the stress diagram of the three-hinged arch with the wind loads as shown in the truss diagram. Fig. 75 (*a*). In order to draw

the polygon of external forces, beginning with point *a* draw the load line *ab*, *bc*, *cd*, *de*, *ef*, *fg*, *gh* and *h—ijklmnop*, Fig. 75 (*b*). The resultant of the wind load, 12,000#, on the vertical portion of the truss has the same line of action as the force *BC*. Likewise the resultant of the wind loads on the sloping part of the upper chord, 24,000#, has the same line of action as the force *FG*. Select a pole and draw the rays numbered 1, 2 and 3. Since the rays 1 and 3 hold the resultant of all the wind loads in equilibrium, the intersection of the sides 1 and 3 of the funicular polygon, shown above the truss

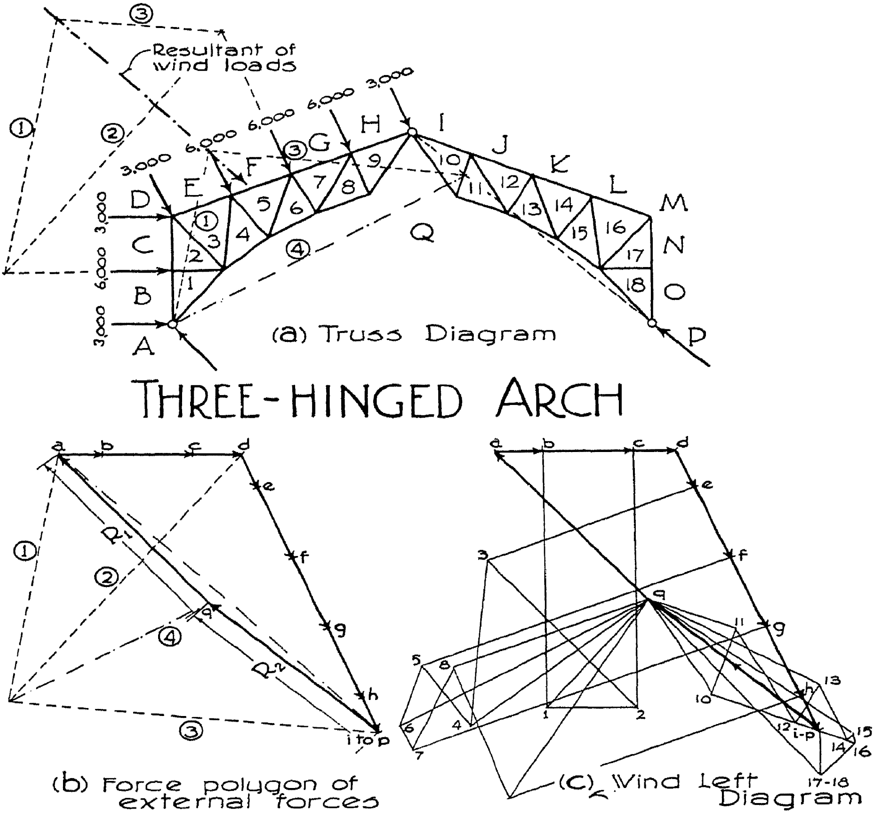


FIG. 75

diagram, determines a point through which the resultant must pass. Its direction and magnitude are given in the force polygon, Fig. 75 (*b*). Therefore, draw a line parallel to the resultant wind load, *a—ijklmnop*, through the point of intersection of the sides of the funicular polygon 1 and 3.

We may now consider the three external forces, R_1, R_2 and the resultant wind load. Of the left reaction, R_1, we know only a point in its line of action, that is, through the hinge at the left support. Of the resultant wind load we know its magnitude, direction and line of action. As for the right reaction, R_2, we know its line of action is parallel to a line drawn through the hinge at the right support and the hinge at the crown of the arch, as was explained in Art. 91. To construct a funicular polygon for these three forces, begin by drawing the string 1, parallel to the ray 1, through the hinge at the left support. From the point where this side of the funicular polygon intersects the resultant of the wind loads draw the string 3 extending it until it meets the line of action of the reaction R_2. The closing string of the funicular polygon is marked number 4, drawn from the intersection of string 3 and the line of action of R_2 to the hinge at the left support. In the force polygon, Fig. 75 (*b*), draw from the pole a line parallel to the closing string 4. Finally, a line drawn from point *ijklmnop*, parallel to the line of action of R_2, intersects the ray 4 at *q*, thus establishing the right-hand reaction R_2. The line from point *q* to point *a* is the reaction R_1 and thereby completes the polygon of external forces.

To avoid a confusion of lines, the polygon just completed is redrawn in Fig. 75 (*c*) and the stress diagram is developed in the usual manner.

CHAPTER 10

MAXIMUM STRESSES

93. **Maximum Stresses.** In the two previous chapters, methods of drawing stress diagrams for various types of trusses and loads have been explained. The purpose of the stress diagram is to determine the character and magnitude of the stresses in the truss members. The loads on a truss are the dead load, snow load and wind load. It is unnecessary to draw a separate stress diagram for the snow loads since, like the dead loads, they are vertical. Having drawn the stress diagram for the dead loads on the upper chord, the stresses for the snow loads may be found directly by proportion.

When the stresses due to dead, snow and wind loads have been established, we must determine the maximum stresses that the truss members will be required to resist. The dead load will always be present, but it is improbable that the maximum wind load and maximum snow load will occur simultaneously. The minimum snow and wind loads are assumed to be equal to one-half their maximum values. To find the maximum stresses in members the following combinations may be considered:

1. Dead-load stresses plus maximum snow-load stresses.
2. Dead-load plus maximum wind plus minimum snow-load stresses.
3. Dead-load plus minimum wind plus maximum snow-load stresses.

When a roller occurs at one support of a truss, separate stress diagrams for wind right and wind left must be drawn. In this case the following combinations may be taken:

1. Dead-load plus maximum snow-load stresses.
2. Dead-load plus minimum snow plus maximum wind-left stresses.
3. Dead-load plus minimum snow plus maximum wind-right stresses.

The greatest stresses resulting from these combinations may be used in designing the truss members. For the larger trusses it is imperative that the various combinations be considered in obtaining maximum stresses. However, the stresses obtained by assuming equivalent vertical loads to replace combined wind and snow loads, see Art. 54, are sufficiently accurate for the more usual types of trusses when no rollers are used. To illustrate these two methods of determining maximum stress the following examples are given.

94. **Maximum Stresses by Various Combinations.** Let it be required to determine the maximum stresses in the Fink truss shown in Fig. 76 (*a*). The panel loads are as follows: dead load, 3,800#; snow load, 4,200 ; wind load, 4,500#. The truss is fixed at both ends, the assumption being that the reactions due to wind loads are parallel.

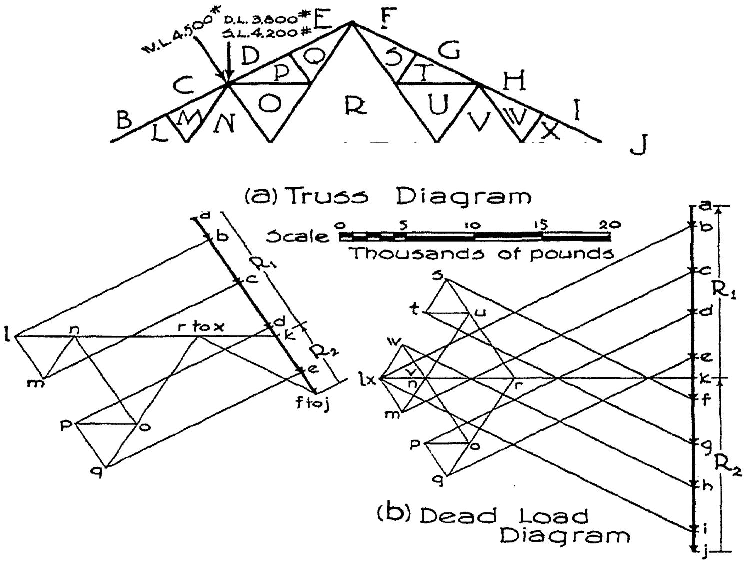


Fig. 76

The stress diagram for the dead loads is first drawn, Fig. 76 (*b*), and the stresses in the truss members are tabulated in Column A, Table XI. A separate stress diagram might be drawn for the snow

Table XI. Stresses in Fink Truss, Fig. 76

In Units of 1,000#

Member	A	B	C	D	E	Combination $A+B$	Combination $A+C+D$	Combination $A+B+E$	Maximum stress
	Dead load	Snow load		Wind load					
		Max.	Min.	Max.	Min.				
BL.....	+26.6	+29.4	+14.7	+16.8	+8.4	+56.0	+58.1	+64.4	+64.4
LK.....	−23.0	−25.4	−12.7	−19.4	−9.7	−48.4	−55.1	−58.1	−58.1
etc.....									

loads. Its shape, however, would be similar to that drawn for the dead loads, and the stresses due to snow loads are directly proportional to the dead-load stresses. For instance, by scaling the length of the line *bl* in the stress diagram we find the stress in member *BL* to be 26,600# due to the dead load. Note that the panel loads are 3,800# and 4,200# for the dead and snow loads respectively. Therefore, the stress in member *BL* due to the snow load is $\frac{42}{38} \times 26{,}600$# or 29,400#. The stresses due to snow loads are tabulated in Column B, Table XI. Since the minimum stresses for snow loads are assumed to be one-half the maximum values, the values in Column B are divided by two and tabulated in Column C.

A separate stress diagram, Fig. 76 (*c*), is drawn for the wind load and the stresses are tabulated in Column D. Minimum wind-load stresses are one-half the maximum values and are listed in Column E.

Referring to Table XI, the three most probable combinations of stresses are shown. For the member *BL*, the combination $A + B + E$ gives the greatest stress, 64,400#. Because of the wind load, the stresses in each side of the truss are not necessarily equal, hence it is necessary to tabulate the stresses in all the truss members. When this has been accomplished, the stresses in members similarly situated in each side of the truss are compared. The stress used in designing the member is the greater of the two. For instance, the maximum stress in member *IX* is 61,200# and, as has been seen, the maximum stress in *BL* is 64,400#. Therefore, the part of the upper chord nearest the support must be designed to resist a compressive stress of 64,400#.

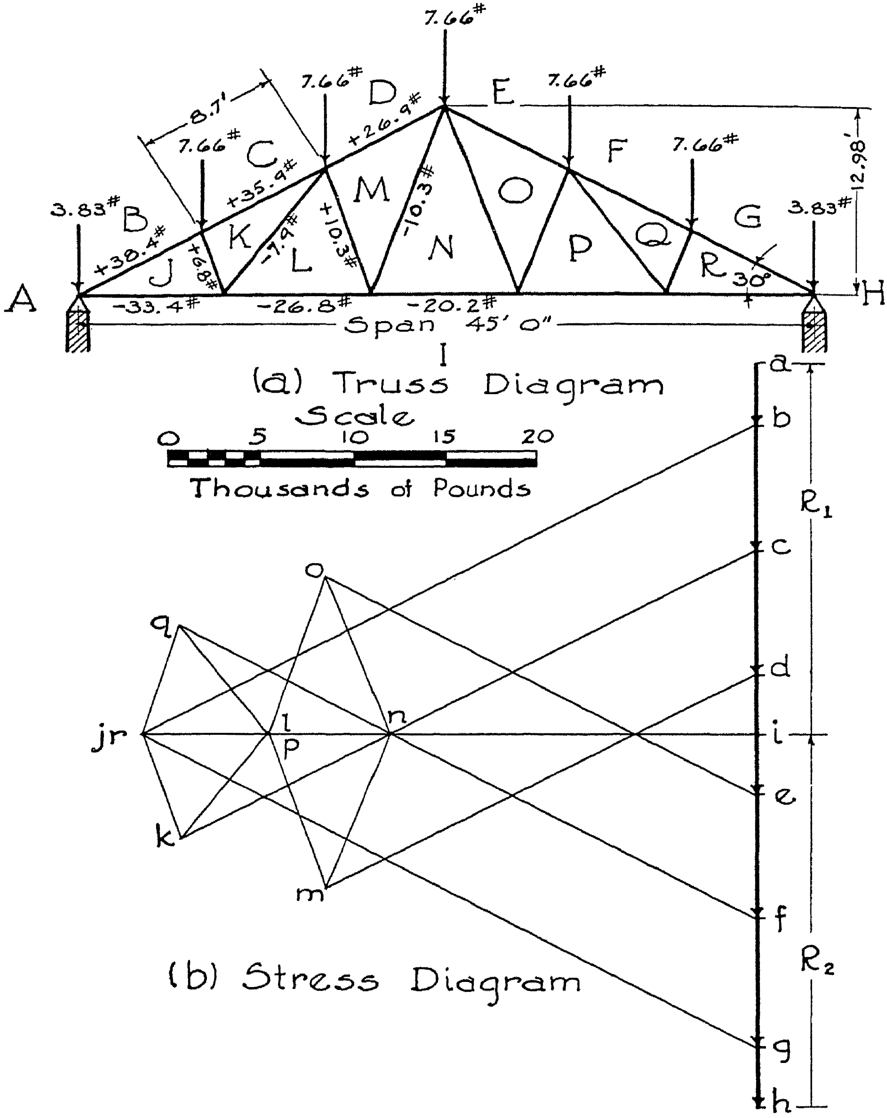

7.66#
7.66#
D
E
7.66#
8.7'
+26.9#
7.66#
C
F
7.66#
+35.9#
M
12.98'
3.83#
B
-10.3#
O
G
3.83#
K
+10.3#
Q
+38.4#
-7.9#
N
P
+6.8#
J
L
R
30°
A
-33.4#
-26.8#
-20.2#
H
Span 45' 0"
I
(a) Truss Diagram
Scale
0
5
10
15
20
Thousands of Pounds
a
b
R1
c
o
q
d
jr
l
p
n
i
e
k
m
f
(b) Stress Diagram
R2
g
h

FIG. 77

95. **Maximum Stresses by Equivalent Vertical Loads.** The method of assuming a vertical load, equivalent to the combined snow and wind loads, necessitates but one stress diagram and, although not so accurate as the preceding method, gives sufficiently accurate stresses for the commonly used trusses. See Art. 54.

Let it be required to determine the stresses in the steel truss shown in Fig. 77 (*a*). The truss has a span of 45′0″ and a pitch of 30°, the trusses being spaced 16′6″ on centers. Since the distance between panel points on the upper chord is 8.7′, 8.7 × 16.5 = 143.5ft², the number of square feet of roof surface transferring loads to the truss at the panel points.

The panel loads are computed as follows:

Flat tile roofing	= 14.0	Table II
1″ hemlock sheathing	= 3.0	Table II
2″ x 6″ rafters, 16″ o.c.	= 1.4	Table III
8″ x 12″ purlins	= 4.0	Approximated
Truss, steel	= 7.0	Table V
Equivalent vertical load for snow and wind combined	= 24.0	Table VIII
Total	= 53.4#/ft.²	

Total vertical panel load = 143.5 × 53.4 = 7,663#

Table XII. Stresses in Truss, Fig. 77

In Units of 1,000#

Member	Character of stress	Magnitude of stress
BJ	Compression	+38.4
CK	Compression	+35.9
DM	Compression	+26.9
JI	Tension	−33.4
LI	Tension	−26.8
NI	Tension	−20.2
JK	Compression	+ 6.8
LM	Compression	+10.3
KL	Tension	− 7.9
MN	Tension	−10.3

In using this method all the loads are assumed to be vertical, hence only one stress diagram is required. This is drawn as shown in Fig. 77 (*b*). In determining maximum stresses in members, it is often convenient to designate the loads and stresses in units of 1,000 pounds, or kips. Note that this has been followed both in Fig. 77 and in Table XII. In the table, only the stresses in one-half the truss are tabulated since those on the opposite side are similar. The character and magnitude of the stresses are noted on the truss members, but a tabulation of stresses is advisable for in the actual design of trusses a table similar to Table XII is prepared and an extra column provided to show the cross section of the member that is to be used.

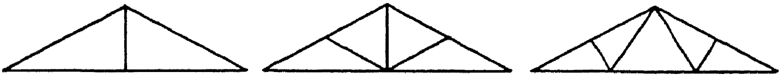
PART III

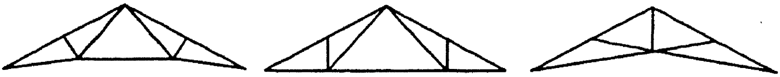
DESIGN OF ROOF TRUSSES

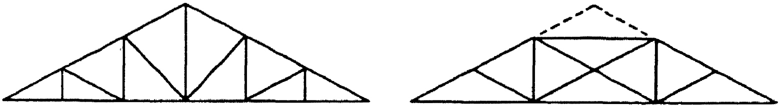
CHAPTER 11

TIMBER TRUSSES

96. **Timber Trusses.** In recent years steel has largely supplanted timber for use in constructing trusses. However, in localities where steel is not readily obtainable, timber is still used extensively. Laminated members and metal connectors permit the use of timber for relatively long spans, 100′ or more, but in general, spans of 30′ to 60′ are the most common.


(a) "A" Truss (b) King Post Truss (c) Simple Fink Truss


(d) Cambered Fink Tr (e) Simple Pratt Truss (f) Scissors Truss


(g) Triangular Howe Truss (h) Queen Rod Truss

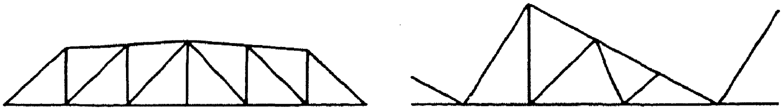

(i) Flat Truss (j) Saw-tooth Truss

FIG. 78

Some of the most usual types of timber trusses are shown in Fig. 78. For the average pitch, the Howe truss, Fig. 78 (*g*), invariably proves to be most economical. Timber readily lends itself to use for compression members but owing to the unavoidable cutting of material at the joints, steel or wrought-iron rods are commonly used for members in tension. For the commonly used trusses, the lower chord is in tension and timber is employed to facilitate the detailing of joints, but the web members resisting tensile forces are more conveniently made of steel or wrought-iron rods.

97. **Allowable Stresses in Timber.** The two woods most frequently used for the construction of trusses are long-leaf yellow pine in the East and Douglas fir in the Western states. Many other woods are used, depending on the proximity to the source of supply. At one time oak was used extensively but its cost is prohibitive today. Table XIII gives the allowable unit stresses for certain timbers. For members in *direct tension,* the same allowable unit stresses are used as those listed for "extreme fiber in bending." The values given in the table may be used with safety, but the

Table XIII. Allowable Working Stresses for Timber*

Species	Grade	Allowable unit stress in pounds per square inch				Modulus of elasticity, pounds per square inch
		Extreme fiber in bending and direct tension	Maximum horizontal shear	Compression		
				Parallel to grain	Perpendicular to grain	
Douglas fir, Coast region	Super Structural and Dense Structural.....	1,800	105	1,300	345	1,600,000
	Structural............	1,600	90	1,200	345	1,600,000
	Common Structural....	1,200	84	1,100	325	1,600,000
Southern yellow pine	Extra Dense Select Structural...........	2,300	200	1,600	475	1,600,000
	Select Structural.......	2,000	175	1,450	375	1,600,000
	Extra Dense Heart.....	2,000	175	1,450	475	1,600,000
	Dense Heart..........	1,800	150	1,300	375	1,600,000
	Structural Square Edge and Sound...........	1,600	125	1,200	375	1,600,000
	No. 1 Common........	1,200	100	1,000	325	1,600,000
West coast hemlock	Select Structural.......	1,300	75	900	300	1,400,000
	Common Structural....	1,040	60	720	300	1,400,000
Oak, commercial, white and red	Select Structural.......	1,400	125	1,000	500	1,500,000
	Common Structural....	1,120	100	800	500	1,500,000

* Compiled from data published by The National Lumber Manufacturers Association.

designer should, of course, consult his local building code to obtain the specific allowable stresses permitted in his community.

98. **Timber Compression Members.** Compression members or struts are designed as columns, assuming the load to be axial and the length of the member to be the distance from center to center of joints. The formula used in the design of compression members will be, of course, the formula specified in the local building code. In the absence of such regulations the following formula may be used with safety:

in which f = the maximum permissible unit stress, in pounds per square inch, for the column cross section

C = the maximum allowable unit stress parallel to the grain for short blocks, in pounds per square inch (This value depends upon the kind of timber to be used. See Table XIV.)

l = the unsupported length of the column, in inches

d = the least width or diameter of the cross section, in inches

In practice, the proper size of the column may be selected directly by referring to a table giving safe axial loads, such as Table XIV. In the absence of such a table, the following steps may be used in the design of timber columns:

STEP 1. Assume a trial cross section of approximately the proper dimensions.

STEP 2. Compute the value of $\frac{l}{d}$ to see that it does not exceed 30, the limit set by many building codes.

STEP 3. Compute the value of f in the column formula.

STEP 4. Multiply the value of f, found in Step 3, by the number of square inches in the cross section of the column. This product is the allowable or safe load on the column that has been assumed.

STEP 5. Compare the allowable load found in Step 4 with the actual load to be supported, sometimes called the design load. If

Table XIV.* Safe Axial Load in Kips for Timber Columns

Nominal size In	Actual size In	Area In^2	Length feet	Douglas fir and long-leaf yellow pine $C = 1,100$	Oak $C = 900$	Short-leaf yellow pine $C = 800$	Norway and white pine $C = 700$	Hem-lock $C = 500$
6 x 6	5½ x 5½	30.3	8	26.0	21.2	18.9	16.5	11.8
			10	24.2	19.8	17.6	15.4	11.0
			12	22.4	18.4	16.3	14.3	10.2
6 x 8	5½ x 7½	41.3	8	35.4	29.0	25.8	22.6	16.1
			10	33.0	27.0	24.0	21.0	15.0
			12	30.5	25.0	22.2	19.4	13.9
			14	28.1	23.0	20.4	17.9	12.8
6 x 10	5½ x 9½	52.3	8	45.0	36.7	32.6	28.6	20.4
			10	41.9	34.2	30.4	26.6	19.0
			12	38.6	31.6	28.1	24.6	17.5
			14	35.5	29.0	25.8	22.6	16.1
6 x 12	5½ x 11½	63.3	8	54.4	44.5	39.5	34.6	24.7
			10	50.6	41.4	36.8	32.2	23.0
			12	46.8	38.2	34.0	29.8	21.2
			14	43.0	35.2	31.2	27.4	19.5
8 x 8	7½ x 7½	56.3	8	52.0	42.5	37.8	33.0	23.6
			10	49.5	40.5	36.0	31.5	22.5
			12	47.0	38.4	34.2	29.9	21.4
			14	44.5	36.4	32.4	28.4	20.2
			16	42.0	34.4	30.6	26.8	19.1
8 x 10	7½ x 9½	71.3	8	65.9	53.9	47.9	41.9	29.9
			10	62.8	51.3	45.6	39.9	28.5
			12	59.5	48.7	43.3	37.9	27.0
			14	56.5	46.2	41.0	35.9	25.6
			16	53.3	43.6	38.8	33.9	24.2
8 x 12	7½ x 11½	86.3	8	79.8	65.2	58.0	50.7	36.2
			10	76.0	62.1	55.2	48.3	34.5
			12	72.1	59.0	52.5	45.9	32.8
			14	68.4	55.9	49.7	43.5	31.0
			16	64.5	52.8	46.9	41.0	29.3
10 x 10	9½ x 9½	90.3	8	86.8	71.0	63.1	55.2	39.4
			10	83.6	68.4	60.8	53.2	38.0
			12	80.1	65.5	58.3	51.0	36.4
			14	77.2	63.3	56.2	49.2	35.1
			16	74.3	60.8	54.0	47.3	33.8
10 x 12	9½ x 11½	109.3	8	105.0	86.0	76.4	66.9	47.8
			10	101.2	82.9	73.6	64.5	46.0
			12	97.1	79.5	70.6	61.8	44.1
			14	93.6	76.6	68.1	59.6	42.5
			16	90.0	73.5	65.4	57.2	40.8
12 x 12	11½ x 11½	132.3	8	130.5	106.9	95.0	83.1	59.4
			10	127.0	103.8	92.2	80.7	57.6
			12	123.0	100.7	89.4	78.2	55.9
			14	119.0	97.4	86.5	75.7	54.1
			16	115.2	94.4	83.9	73.4	52.4

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* Taken from Kidder-Parker, *Architects' and Builders' Handbook*, John Wiley & Sons, New York.

the allowable load on the trial section is less than the design load, a larger cross section is assumed and again tested in the same manner.

To illustrate the above procedure, let it be required to design a long-leaf yellow pine column, 10′0″ in length, having an axial load of 80,000#.

Step 1. Assume a trial section having nominal dimensions of 10″ x 10″. The actual dimensions for this nominal size are 9.5″ x 9.5″. See Table XIV.

Step 2. The ratio $\frac{l}{d} = \frac{10 \times 12}{9.5} = 12.6$. This falls within 30, the limiting ratio.

Step 3. $f = C\left(1 - \frac{l}{80d}\right)$. In this equation C will equal 1,100#/in². See Table XIV.

Then $f = 1{,}100\left(1 - \frac{10 \times 12}{80 \times 9.5}\right)$ and $f = 926.32\#/\text{in}^2$.

Step 4. The actual dimensions of the cross section are 9.5″ x 9.5″, therefore the area is 90.25in². The allowable load on the trial section hence is

$$90.25 \times 926.32\# = 83{,}600\#$$

Step 5. Since the design load on the column is only 80,000# and the safe load on the section that has been assumed is 83,600#, the 10″ x 10″ column is acceptable.

Note that this column might have been selected from Table XIV without computations.

99. Timber Tension Members. All tension members formed of timber have their sections reduced at joints by the necessary cutting for bolts and framing. Therefore, in designing timber tension members the cross section must have a gross area in excess of the required net area. In timber trusses it is customary to use timber for the lower chord which resists tensile forces, the gross section being one-half to two-thirds greater than the net area theoretically required. To find the net area all that is necessary is to divide the tensile force to be resisted by the allowable tensile unit stress of the timber. The allowable tensile unit stresses are the same as given for "extreme fiber in bending" in Table XIII.

When the joints are detailed, the area to be cut away may be computed and the remaining portion checked against the required net area.

To illustrate, let it be required to determine the size of a timber member to resist a tensile force of 38,000#. The timber will be longleaf southern yellow pine, No. 1 common grade.

Referring to Table XIII, the allowable tensile unit stress for this timber is 1,200#/in^2. Therefore, $38{,}000 \div 1{,}200 = 31.66$in^2, the required net area. Assuming the gross area to be two-thirds greater than the net area, $31.66 \times 5/3 = 52.76$in^2, the section required. A section having nominal dimensions of 8″ x 8″ has actual dimensions of 7.5″ x 7.5″ or 56.25in^2 and this section is acceptable. For convenience in framing, it is customary to use upper and lower chords of the same width. A 6″ x 10″ cross section has an actual cross section of 52.25in^2 and this section might also be used, if preferred.

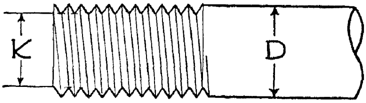
100. **Steel Rods Used for Tension Members.** Web members resisting tensile forces in timber trusses are commonly made of steel rods. The net area of a rod is its area at the root of the thread as given in Table XV. The usual allowable tensile unit stress for steel rods is 16,000#/in^2 or 18,000#/in^2. The net area of the rod multiplied by the allowable unit tensile stress determines the safe tensile load the rod will resist. Table XV gives the gross and net areas of rods commonly used and also their safe tensile loads based on net areas and a unit stress of 16,000#/in^2. Having determined the force to be resisted, the proper size of the rod may be determined by selecting it directly from the table.

When trusses are constructed, an *initial tension* is set up in the rods by the tightening of nuts and the possible distortion of various members. An allowance of 1,000# to 3,000#, depending on the size of the rod, is made for this indeterminate stress. To determine the size of the rod to use, add the allowance for initial tension to the tensile force to be resisted. This sum divided by the allowable tensile unit stress gives the required net area and may be selected by referring to Table XV.

Let it be required to determine the size of a steel rod in a truss to resist a tensile force of 12,000#.

Assuming the initial tension to be 1,500, $12{,}000 + 1{,}500 = 13{,}500$#, the theoretical tensile force to be resisted. Referring to

Table XV. Safe Tensile Loads for Round Steel Rods



Diameter		Area		Safe tensile load based on area at root of thread at 16,000#/in^2
Total D In	Net *K* In	Total Diam. D In^2	Net Diam. *K* In^2	
½	0.400	0.196	0.126	2,016
⅝	0.507	0.307	0.202	3,232
¾	0.620	0.442	0.302	4,832
⅞	0.731	0.601	0.419	6,704
1	0.838	0.785	0.551	8,816
1⅛	0.939	0.994	0.693	11,088
1¼	1.064	1.227	0.890	14,240
1⅜	1.158	1.485	1.054	16,864
1½	1.283	1.767	1.294	20,704
1⅝	1.389	2.074	1.515	24,240
1¾	1.490	2.405	1.744	27,904
1⅞	1.615	2.761	2.049	32,784
2	1.711	3.142	2.300	36,800
2¼	1.961	3.976	3.021	48,336
2½	2.175	4.909	3.716	59,456
2¾	2.425	5.940	4.619	73,904
3	2.629	7.069	5.428	86,848

Table XV, we find that a steel rod having a diameter of 1¼″ will resist a safe load of 14,240# and is, therefore, acceptable. Note that this safe load is based on the area at the root of the thread and that the allowable tensile unit stress is 16,000#/in^2.

101. **Framing of Joints.** The first step in making working drawings for a truss is to make a diagram of the *working lines*. The working lines correspond to the truss diagram, the system of lines representing the truss by means of which the stress diagram is developed. These lines intersect at common points at the joints. After the sizes of the members have been determined, the members are laid out over the working lines, *the center-of-gravity lines of the members coinciding with the working lines.* The working lines for the accompanying details are indicated by dot-and-dash lines.

In designing a timber truss care in detailing the joints is of the utmost importance. In general, the joints should be as simple as possible with a minimum cutting of material. Owing to the inherent shrinkage of timber, bolts and rods should be retightened a reasonable period after erection.

102. **Heel Joints.** One of the most critical joints in a timber truss is the intersection of the upper and lower chords at the supports. This is sometimes known as the heel joint. Regardless of the type of joint, timber trusses supported on masonry walls should rest upon a steel plate or masonry block so that the load from the truss is distributed over an adequate area. The necessary cutting of the upper chord is of minor importance as compared with that of the lower chord which, of course, resists a tensile force. If this member is reduced excessively, a supplementary timber, called a *bolster*, Fig. 79 (*a*), may be added to provide additional area. In addition to the tensile force to be resisted by the lower chord, provision must be made for longitudinal shear. This may be accomplished by the use of bolts, cast-iron shoes, steel plates, etc., as suggested by the diagrams in Fig. 79.

To illustrate the forces that must be held in equilibrium, the following example is given.

The upper and lower chords at the heel of a truss consist of 6″ x 6″ long-leaf yellow pine timbers of structural grade. The upper chord resists a compressive force of 14,000#, and 12,100# is the tensile force to be resisted by the lower chord. Let it be required to detail this joint.

Two methods will be explained, the first of which is based on the assumption that the timber of the lower chord resists the entire tendency to fail by horizontal shear. Referring to Fig. 80 (*a*), the surface designated "*A*" represents the surface which resists this horizontal shear. A force diagram is drawn, Fig. 80 (*b*), showing that a force of 14,000# has horizontal and vertical components of 12,100# and 700# respectively. Therefore, 12,100# is the force to be resisted by horizontal shear in the surface "*A*." A timber having nominal dimensions of 6″ x 6″ has actual dimensions of 5.5″ x 5.5″. Hence, the beam being 5.5″ in width, the problem resolves itself into determining the length of the surface "*A*." Table XIII shows the allowable horizontal shearing unit stress to be $125\#/\text{in}^2$. Then $12{,}100 \div 125 = 96.8\text{in}^2$, the required area of surface "*A*." The beam being 5.5″ in width, 96.8 ÷ 5.5 = 17.6″, the minimum length of surface "*A*" to resist the horizontal shearing stress. Call this distance 18″, Fig. 80 (*a*).

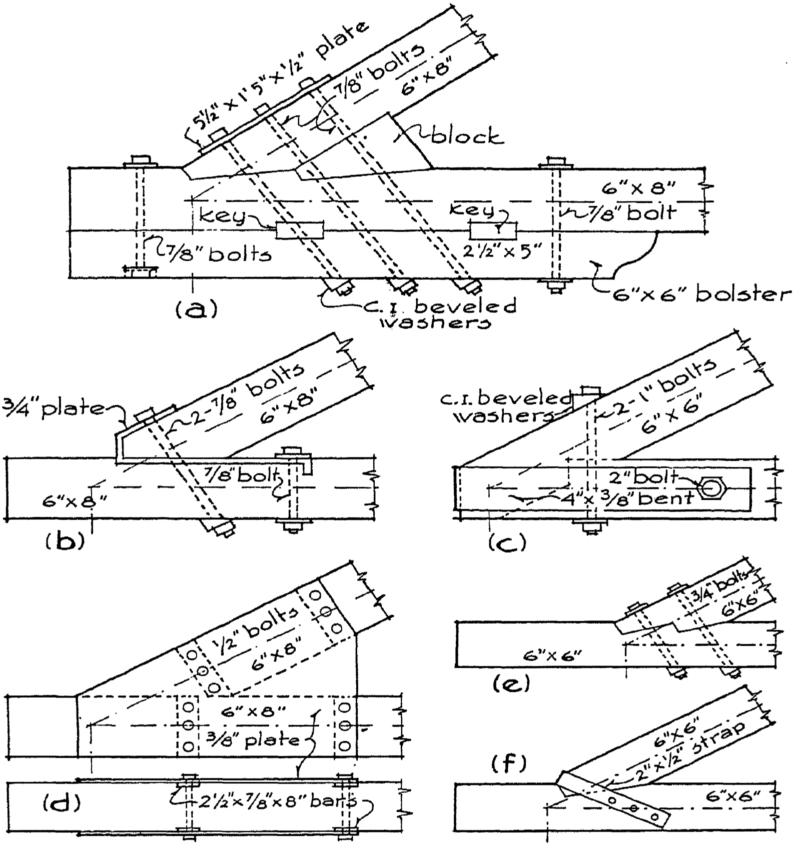


FIG. 79

The next step is to determine the depth of surface "*B*," the end of the upper chord cut at a right angle with the direction of the member. The upper chord makes an angle of 30° with the horizontal, hence the angle between the inclined surface and the direction of the grain of the wood in the lower chord is 60°. When the bearing surface of a compression member makes an angle with the fibers of the timber, the allowable bearing unit stress may be found by referring to the graph shown in Fig. 81, the vertical distances giving the allowable bearing stresses, the horizontal distances indicating the various angles. Since, in this instance, the timber is long-leaf yellow pine and the angle is 60°, the allowable compressive unit stress is 1,130#/in². Therefore, 14,000 ÷ 1,130 = 12.3in², the minimum area required. The member being 5.5″ in width, 12.3 ÷ 5.5 = 2.23″, say 2¼″, the depth of surface *"B."*

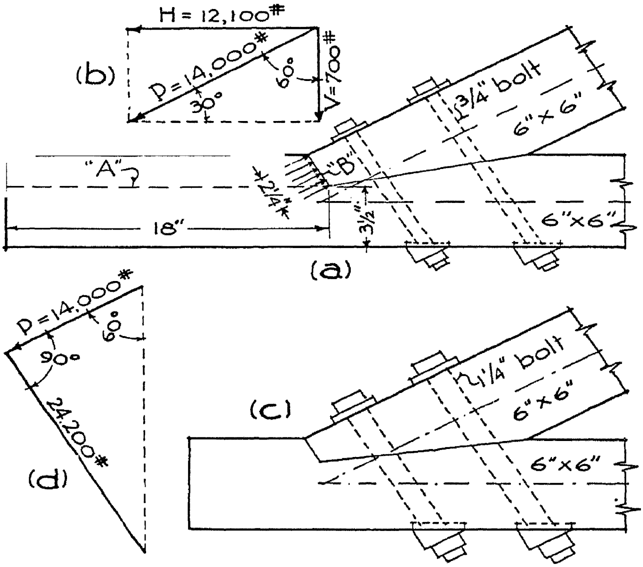


Fig. 80

The lower chord has now been reduced so that from a gross area of 5.5″ x 5.5″, the net area is only 3.5″ x 5.5″ or 19.25in². If the lower chord fails by tension it will tear apart at this reduced area. Let us investigate this member for tension. Since the tensile force to be resisted is 12,100# and the net area is 19.25in², 12,100 ÷ 19.25 = 628.5#/in², the actual unit tensile stress. As this stress is less than 1,600#/in², the allowable tensile unit stress given in Table XIII, the member will safely resist the tensile force of 12,100#.

Note that the allowable tensile unit stress is similar to the "extreme fiber in bending" as given in the table.

For practical considerations two ¾" bolts are used to secure the upper and lower chords, but the above computations are based on the assumption that the timber resists the entire thrust. The disadvantage of the method just described is that the projection of the lower chord may extend beyond the exterior face of the wall, thus making the joint impracticable. In order to overcome this objection, the joint may be designed so that bolts or straps resist the entire thrust, not depending on the shearing stresses in the timber.

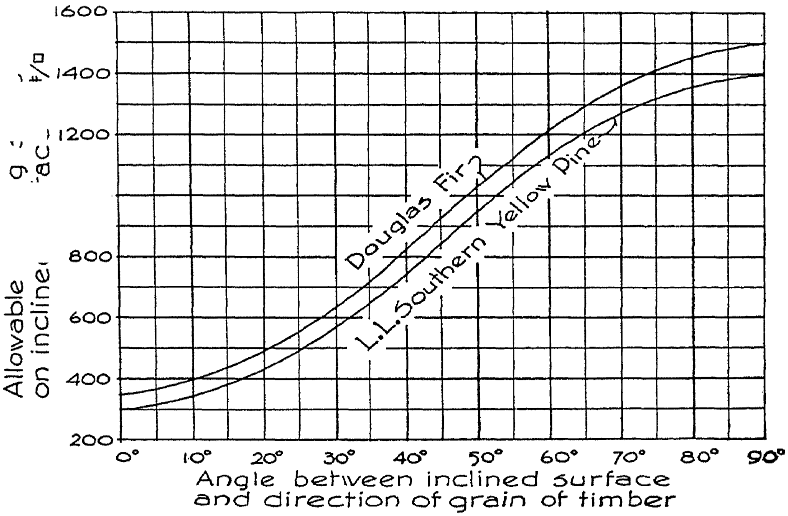


FIG. 81

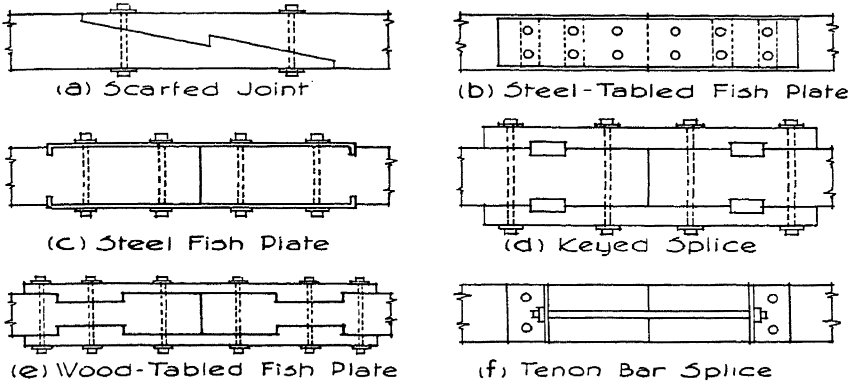
For computations based on this assumption, it is unnecessary to notch the lower chord, but since such a detail is an aid in erection, the upper chord is let into the lower chord slightly to keep it in position while the members are being fitted and the holes bored.

A force polygon, Fig. 80 (*d*) is again drawn and the tension to be resisted by the bolts is found graphically to be 24,200#. Since two bolts will be used, each bolt must resist one-half of this value or 12,100#. As explained in Art. 100, assume the initial tension to be 1,000#, thus making the tensile force to be resisted by one bolt 12,100 + 1,000 or 13,100#. Referring to Table XV it is found that a steel bolt having a diameter of 1¼″ will safely resist 14,240# and therefore is acceptable. Two 1¼″ bolts will be used as shown in Fig. 80 (*c*). The horizontal component of the force 14,000# is 12,100# as shown in Fig. 80 (*b*). This tendency to shear horizontally must be resisted by the timber between the let-in washers on the under side of the beam and the exterior end of the lower chord. As found in the preceding case this distance must be at least 18″. The advantage of this joint lies in the fact that the projection of the lower chord may be made shorter than is required by the detail shown in Fig. 80 (*a*).

As actually constructed, both the timber and bolts share in resisting the thrust. However, since the timber and steel seldom act in unison, the joint should be so designed that either one or the other resists the entire thrust without relying on the two acting jointly.

103. **Tension Splices.** Owing to the difficulty in obtaining a single timber of sufficient length for the entire lower chord for timber trusses, it is often necessary to splice two members together. For the common types of triangular trusses, the stresses in the lower chord members at the center of the span are a minimum at this point, consequently it is advisable to make the splice near the center. Various types of tension splices are shown in Fig. 82. Probably the most common splice is known as the fish-plate splice which consists of either timber or steel plates bolted to each side of the lower chord. When steel plates are used, rectangular steel strips called *tables* may be let into the timber on each side, flush with the surface and covered by the steel splice plates. When timber splice members are employed, their net section must be at least equal to that of the main member. The bolts securing the members are subjected to bending and shearing stresses and sufficient timber must be provided between the bolts to resist longitudinal shear.

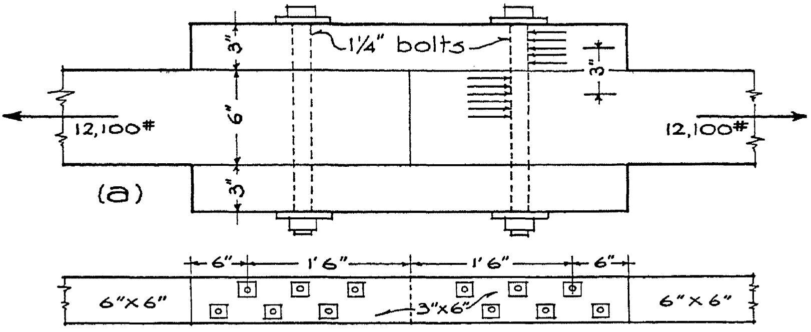
To illustrate the computations required in designing a timber fish-plate splice, assume the lower chord to consist of a 6″ x 6″, actual-size, long-leaf yellow pine timber of structural grade, the total tensile stress to be resisted being 12,100#. Use steel bolts 1¼″ in diameter and 3″ x 6″ timber pieces on each side, Fig. 83.


(a) Scarfed Joint
(b) Steel-Tabled Fish Plate
(c) Steel Fish Plate
(d) Keyed Splice
(e) Wood-Tabled Fish Plate
(f) Tenon Bar Splice

TENSION SPLICES

FIG. 82

To maintain as great a net section as possible, the bolts will be staggered as shown in Fig. 83 (*b*). The area of the main member



Timber Fish-plate Splice

FIG. 83

to be deducted for the bolt hole will be 6″ × 1.25″ or 7.5in^2; thus the net section will be $(6 \times 6) - 7.5$ or 28.5in^2. The tensile force to be resisted is 12,100#, hence $12{,}100 \div 28.5 = 425\#/\text{in}^2$, the actual tensile unit stress. Since from Table XIII the allowable tensile unit stress is 1,600#/in², the 6″ x 6″ section is acceptable for tension.

Next determine the number of bolts required due to bending. Since the bolts tend to bend on each side of the main chord, the force producing bending will be 12,100 ÷ 2 or 6,050#. See Fig. 83 (*a*). This force is represented by the arrows. The lever arm between the resultants of the two opposite forces being 3″, the bending moment to be resisted by the bolts is 6,050# × 3″ or 18,150″#. The flexure formula* for a member in bending is $\frac{M}{f} = S$. That is to say, the bending moment divided by the allowable extreme fiber stress is equal to the required section modulus. Assuming $f = 16{,}000\#/\text{in}^2$, $18{,}150 \div 16{,}000 = 1.134\text{in}^3$, the required section modulus. The value of S, the section modulus for a circular cross section, is $\frac{\pi d^3}{32}$, hence S for a bolt having a diameter of 1.25″ is $\frac{3.1416 \times 1.25^3}{32}$ or 0.19144in^3. Then $1.134 \div 0.19144 = 5+$, the required number of bolts to resist bending. Use six bolts on each side of the joint.

The allowable unit bearing stress on a bolt (on projected diameter) is the average between the allowable bearing stress parallel with the grain and the allowable bearing stress perpendicular to the grain or $(1{,}200 + 375) \div 2 = 787\#/\text{in}^2$, Table XIII. There being six bolts, the bearing on each bolt will be 12,100 ÷ 6 or 2,017#. The projected area of one bolt is 6 × 1.25 or 7.5in²; $2{,}017 \div 7.5 = 269\#/\text{in}^2$, which is the actual bearing stress on each bolt. Since this value is less than the allowable, 787#/in², six bolts are acceptable.

Testing the bolts for shear, a 1¼″ bolt has an area of 1.227in², and assuming 10,000#/in² to be the allowable shearing unit stress, $1.227 \times 10{,}000 = 12{,}270\#$, the allowable shearing stress for one bolt in single shear. But each bolt to fail by shear must fail at two surfaces, therefore $12{,}270\# \times 2 = 24{,}540\#$, the allowable shearing stress for one bolt in double shear. The total force to be resisted is only 12,100# and as this is to be resisted by six bolts, ample material is provided for shearing in the bolts.

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* See Art. 39, *Simplified Engineering for Architects and Builders.*

There is also a tendency for the timber to fail by horizontal shear at each bolt. For practical considerations it is advisable to space the bolts at least 6″ apart. Then each bolt has two timber shearing surfaces each 6″ x 6″, and since there are six bolts, $(6 \times 6 \times 2 \times 6) = 432\text{in}^2$, the number of square inches of surface effective in resisting longitudinal shear. The total force to be resisted is 12,100# and the allowable horizontal shearing unit stress of this timber is $125\#/\text{in}^2$ (Table XIII). Therefore, $12{,}100 \div 125 = 96.8\text{in}^2$, the minimum required area. Since the 6″ spacing of bolts provides a greater area than is required, the joint is acceptable.

104. **Combined Bending and Direct Stresses.** When loads occur on a truss member between joints, it is necessary to design the member for both a direct stress and the stresses due to bending. Such a condition occurs when purlins are omitted and the roof rafters are supported by the upper chord, or when a ceiling load rests directly upon a lower chord. See Figs. 30 and 84. The following example illustrates the design procedure for this type of loading.

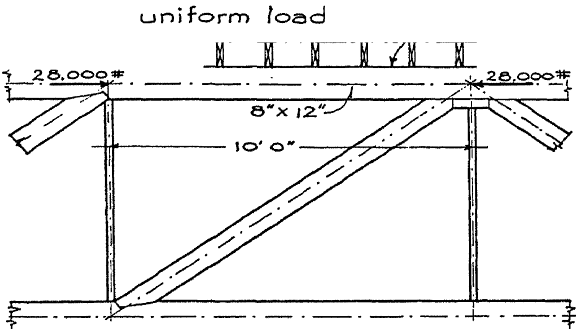


FIG. 84

The upper chord of a truss has a span of 10′0″. This member is subjected to a direct compressive stress of 28,000# and a uniformly distributed load of 600 pounds per linear foot producing bending. The timber is long-leaf yellow pine. Let it be required to design the member. See Fig. 84.

Assume the member to be an 8″ x 12″ with the long axis vertical. The slenderness ratio, $\frac{l}{d} = \frac{10 \times 12}{7.5} = 16$, is acceptable as it is less than 30, the limiting ratio. In accordance with the column formula, Art. 98, the maximum allowable unit stress for this member is determined by the equation $f = 1{,}100\left(1 - \frac{l}{80d}\right)$

or

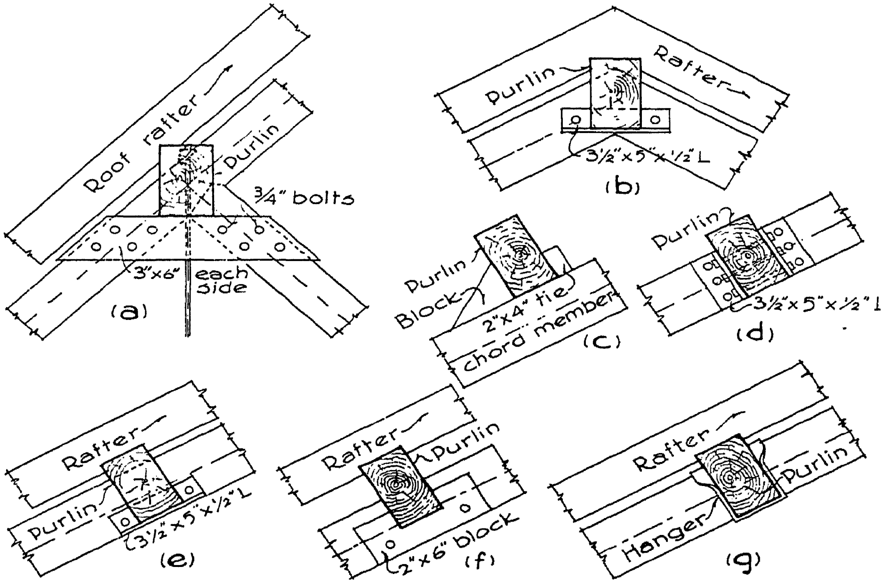
The actual dimensions of an 8″ x 12″ timber are 7.5″ x 11.5″, hence the stress due to the direct compressive force of 28,000# is 28,000 ÷ (7.5 × 11.5) or 325#/in^2.

The next step is to compute the maximum unit stress due to bending. As the upper chord is continuous, assume the maximum bending moment to be $\frac{wl^2}{10}$. Then $M = \frac{600 \times 10 \times 10 \times 12}{10} =$ 72,000″#. The section modulus of a rectangular cross section is $\frac{bd^2}{6}$, hence the flexure formula becomes $\frac{M}{f} = \frac{bd^2}{6}$ or $f = \frac{6 \times M}{bd^2}$. Substituting the known quantities, $f = \frac{6 \times 72{,}000}{7.5 \times 11.5 \times 11.5} =$ 436#/in^2, the extreme fiber stress due to bending. Adding this to the stress due to direct compression, 436 + 325 = 761#/in^2. This unit stress being within the maximum stress of 880#/in^2 permitted by the column formula, the 8″ x 12″ cross section is acceptable.

105. **Timber Purlins.** Purlins are the beams extending from truss to truss which transfer the roof loads to the trusses. Fig. 85 illustrates various types of purlin connections. The most common method employed is to frame the purlin to the truss so that one of its major axes is parallel to the upper chord.

To illustrate the design of a timber purlin, assume the vertical uniformly distributed load, including the weight of the purlin, to be 8,600#, the span between trusses is 16′0″ and the timber is long-leaf yellow pine, No. 1 common grade. The sheathing consists of 3″ planking, Fig. 86 (*a*).

First draw a force polygon, Fig. 86 (*b*), determining graphically the components of 8,600#, one component being parallel and the other normal to the pitch of the roof. These components are 4,300# and 7,500# respectively. In the present instance the plank roofing stiffens the purlin in the direction parallel to the pitch of the roof and, therefore, the component 4,300# is assumed to be resisted by the planking. With this assumption, the purlin is free to bend only in a direction normal to the pitch of the roof, that is, about the axis X-X, and the force producing this bending is the component 7,500#.



PURLIN CONNECTIONS

FIG. 85

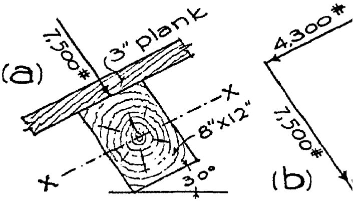


FIG. 86

Since this is a simple beam having a span of 16′0″, the value of the maximum bending moment is $M = \frac{Wl}{8} = \frac{7{,}500 \times 16 \times 12}{8} =$

180,000″#. The flexure formula is $\frac{M}{f} = S$, and as the allowable extreme fiber stress for this timber is 1,200#/in², Table XIII, $\frac{180,000}{1,200} = 150\text{in}^3$, the required section modulus. The section modulus of a rectangular cross section is $\frac{bd^2}{6}$.* Hence, if we assume the width of the beam to be 7.5″,

$$S = \frac{bd^2}{6} \text{ or } 150 = \frac{7.5 \times d^2}{6} \text{ or } d = 10.95''$$

Therefore, a purlin having a nominal size of 8″ x 12″ is acceptable.

106. **Miscellaneous Truss Connections.** In detailing the joints of a timber truss there are numerous types of connections that may be used. It is good practice to cut into the upper and lower chords as little as possible. This is particularly true with respect to the lower chord since cutting reduces the effective area. The upper and lower chords are made of continuous members whenever possible and the design force to be used is the maximum stress in the chord, usually the stress in the member adjacent to the support. For intermediate joints, economy of framing is often effected by designing the connection for the members resisting the greatest stresses and duplicating this joint at the remaining intermediate connections.

Having selected a steel or wrought-iron rod for a member resisting tension, the washer to be used must have an area sufficiently large to provide the necessary bearing on the timber. For the rod shown in Fig. 87 (*i*), the washer produces a compressive force perpendicular to the grain of the timber, the permissible stresses being found in Table XIII. A steel plate is often used in place of the stock cast-iron washer. Beveled washers, as indicated in Fig. 87 (*f*), may be used, or the timber may be cut normal to the direction of the rod as in Fig. 87 (*e*). When the bearing surface is oblique with the grain of the timber, the allowable bearing stresses may be found in Fig. 81.

Much progress has been made in recent years in developing metal *timber connectors*. Connectors are metal devices used for joining structural timbers. They are particularly applicable to trusses and their use has permitted the construction of timber trusses having spans as great as 120′.

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* See Art. 43, *Simplified Engineering for Architects and Builders.*

One type of connector consists of a circular band having teeth on each edge as indicated in Fig. 87 (*m*). When two timbers are to be joined they are placed side by side with the connector embedded for half its depth in each member. The connectors are forced into the timber by the tightening of a bolt which also serves to prevent the members from spreading. The number of connectors to use in a joint is the force to be transmitted divided by the allowable working value of one connector. This type is known as a *toothed-ring connector* and is made in different sizes, 2″ to 4″ in diameter, and a depth of slightly less than 1″.

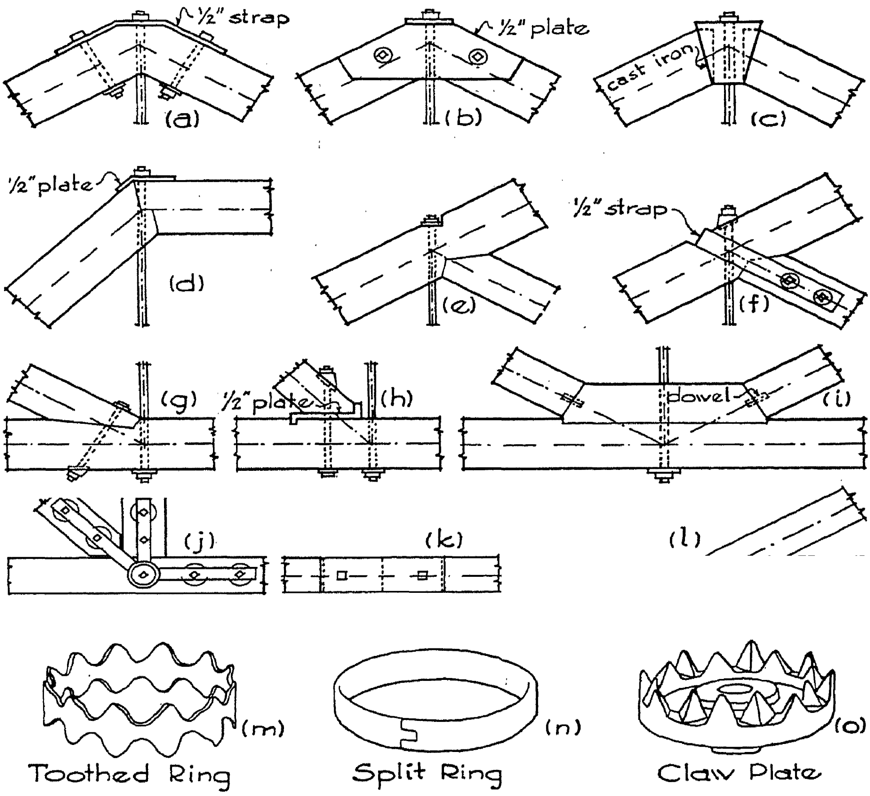

FIG. 87

Another type is known as a *split-ring connector* shown in Fig. 87 (*n*). It is similar to the toothed-ring connector in principle, being split to permit slight variations in diameter, but lacking the teeth on the edges. Split-ring connectors vary from 2½″ to 6″ in diameter. To install, a circular groove is made in each timber into which the ring is set, the timbers being held in contact by bolts at the centers of the rings. See Fig. 87 (*l*).

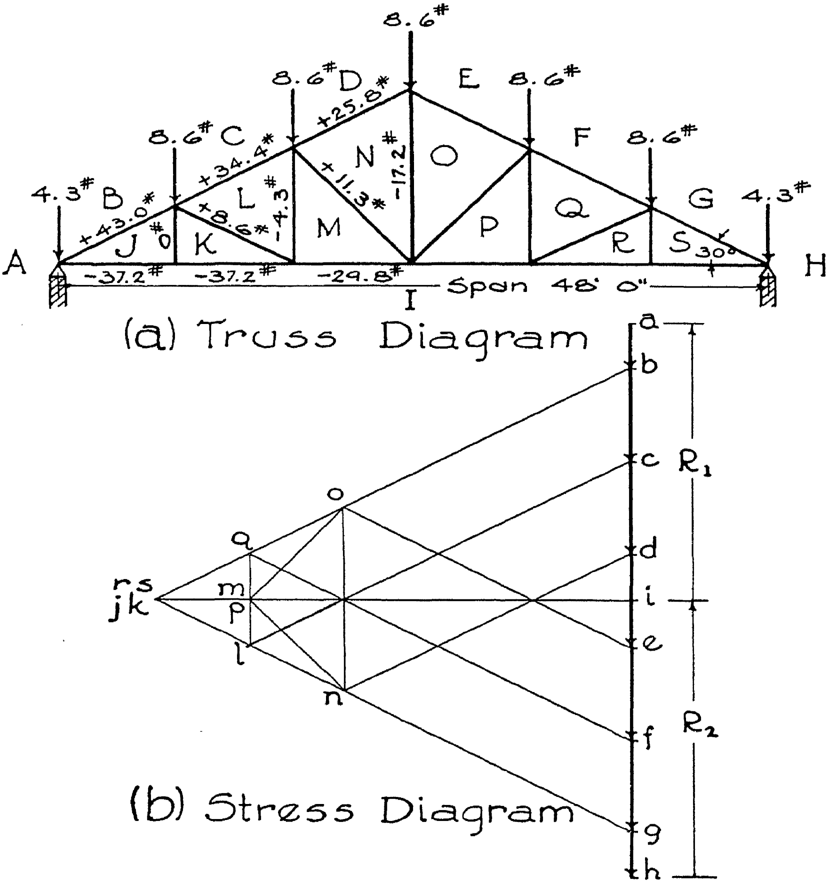

FIG. 88

Fig. 87 (*j*) indicates a connection formed of metal discs with teeth which are forced into the timbers when the bolts are drawn up. A steel strap connects the discs in each member, the number of discs being determined by the magnitude of the stresses in the members. These discs are known as *claw plates*, illustrated in Fig. 87 (*o*).

Another type of connector, used to splice members, consists of a metal plate with teeth at the ends, as indicated in Fig. 87 (*k*).

The great advantage in using the so-called modern timber connector lies in the fact that there is a minimum reduction of cross-section area, resulting thereby in a saving of material.*

107. **Design of a Timber Truss.** The following discussion describes briefly the necessary steps in the design of a typical timber truss.

Let it be required to design a triangular six-panel Howe timber truss in accordance with the following data. The span of the truss from center to center of supports is 48′0″. The pitch of the roof is 30° and the trusses are spaced 16′0″ on centers. The roofing surface is ⅜″ slate on roofing felt laid over 3″ yellow pine sheathing supported on 8″ x 12″ purlins. The truss is to be constructed in one of the Central states and equivalent vertical loads will be employed for combined snow and wind loads. The timber will be Southern yellow pine, No. 1 common grade.

Begin by drawing a truss diagram as shown in Fig. 88 (*a*). The upper chord is divided into three equal lengths of 9′3″ each. Since the trusses are spaced 16′0″ on centers, $9.25 \times 16 = 148\text{ft}^2$, the roof area transferring loads to the truss at the panel points.

Computing the loads per square foot of roof surface,

⅜″ Slate and roofing felt = 15 Table II
3″ Plank sheathing = 12 Table II
Equivalent snow and wind load combined = 24 Table VIII
Truss = 4 Table IV
Total = $55\#/\text{ft}^2$ not including the weight of the purlin

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* Data concerning timber connectors are published by the National Lumber Manufacturers Association, Washington, D. C., and the Forest Products Laboratory, Madison, Wisconsin.

Panel load without purlin = 148 × 55 = 8,140#.

Assuming the weight of timber to be 40#/ft^3, the weight of the 8″ x 12″ purlin, 16′0″ in length, will be

$$\frac{8 \times 12}{144} \times 16 \times 40 = 427\#$$

Then the panel load including the weight of the purlin will be 8,140 + 427 = 8,567#, say 8,600# or 8.6 kips.

In accordance with this panel load, draw a stress diagram as shown in Fig. 88 (*b*). The lengths of the lines in the stress diagram are scaled and the characters and magnitudes of the stresses in the various members are tabulated as shown in Table XVI. It should be noted that, in this instance, the magnitudes of the stresses are determined by means of the stress diagram. However, these stresses might have been found by the use of Table IX, thereby making it unnecessary to construct the stress diagram. Refer to Art. 81 and check these stresses.

Table XVI. Stresses in Truss Shown in Fig. 88

Member	Character of stress	Magnitude of stress in kips	Cross section of member to be used
BJ..........	Compression	43.0	8″ x 8″
CL..........	Compression	34.4	8″ x 8″
DN..........	Compression	25.8	8″ x 8″
JI..........	Tension	37.2	8″ x 8″
KI..........	Tension	37.2	8″ x 8″
MI..........	Tension	29.8	8″ x 8″
KL..........	Compression	8.6	6″ x 8″
MN..........	Compression	11.3	6″ x 8″
JK..........	Zero stress	0.0	½″ diam. rod
LM..........	Tension	4.3	⅞″ diam. rod
NO..........	Tension	17.2	1½″ diam. rod

Having found the stresses, the next step is to determine the size of the various members to resist these stresses. For convenience and economy in framing, both the upper and lower chords will be made of continuous members having uniform cross sections. After the width of the upper chord has been established all the remaining timber members will have this same width regardless of the magnitudes of their stresses.

Member BJ. This member is in compression, having a stress of 43,000# and a length of 9′3″. In accordance with the procedure outlined in Art. 98, the design of this member is accomplished as follows:

Step 1. Assume this member to be an 8″ x 8″ having actual dimensions of 7.5″ x 7.5″ and an area of 56.25in².

Step 2. The slenderness ratio, $\frac{l}{d} = \frac{111}{7.5} = 14.8$. This is acceptable since the limiting ratio is 30.

Step 3. $f = 1{,}100\left(1 - \frac{l}{80d}\right)$ or $f = 1{,}100\left(1 - \frac{111}{80 \times 7.5}\right)$ or $f = 896.5\#/\text{in}^2$.

Step 4. $896.5 \times 56.25 = 50{,}428\#$, the permissible load on the assumed cross section. This load being in excess of 43,000#, the design load, the 8″ x 8″ cross section is acceptable. It is found that a 6″ x 8″, the next smaller stock size, is inadequate.

The other members of the upper chord, *CL* and *DN*, have the same length and the magnitudes of their stresses are smaller, therefore the upper chord will be made of one continuous timber having nominal dimensions of 8″ x 8″. By referring to Table XIV note that member *BJ* might have been selected directly, thereby avoiding the above computations.

Member JI. This member is in tension and the stress to be resisted is 37,200#. The length of a member in tension has no effect on its design. The allowable tensile stress for this timber is 1,200#/in², Table XIII. Therefore, $37{,}200 \div 1{,}200 = 31\text{in}^2$, the required net area. As explained in Art. 99, we will assume the gross area to be two-thirds greater, to provide for the necessary cutting at the joints. Then $31 \times \frac{5}{3} = 51.7\text{in}^2$, the required gross area. An 8″ x 8″ cross section has an actual area of 56.25in² and consequently will be acceptable. The entire lower chord will have this same cross section. Due to its length, it will be necessary to make a splice at the center of the span.

Member KL. The length of this member is 9′3″ and resists a compressive force of 8,600#. As has been stated, this member will be made 8″ in width to conform to the upper and lower chords. A nominal dimension of 4″ is actually 3⅝″, therefore assuming the member to be 4″ x 8″, $\frac{l}{d} = \frac{111}{3.625} = 30.7$, slightly in excess of

30, the limiting ratio. Actually, this size could be used as the 9.3″ dimension is the length of the line in the truss diagram and the true length of the member is somewhat less. The next stock size is 6″ x 8″. Referring to Table XIV it is seen that this size is amply large for the force of 8,600# and, therefore, it will be employed.

Member MN. The member *MN* has a length of 12′3″ and a compressive force of 11,300#. Assuming the member to be 6″ x 8″, $\frac{l}{d} = \frac{147}{5.5} = 26.7$, acceptable since it is within 30, the limiting ratio.

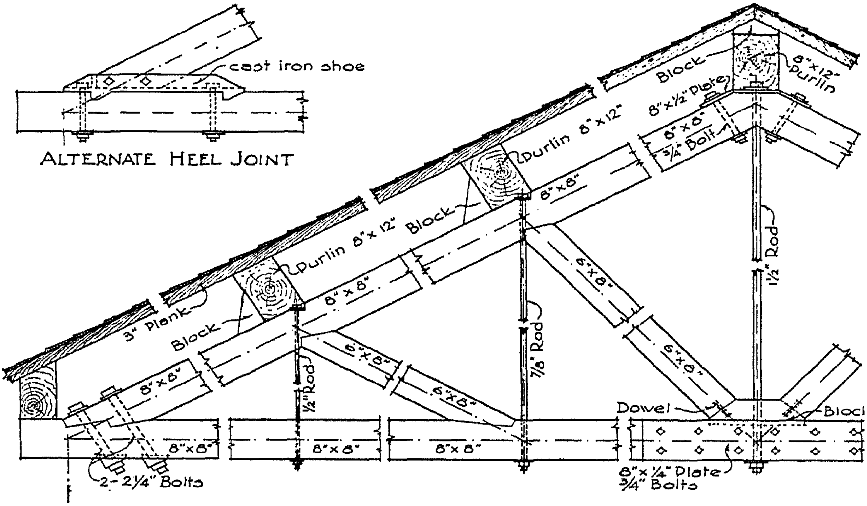

FIG. 89

Table XIV shows that a 6″ x 8″ member is sufficiently large to resist the load of 11,300#.

Member JK. There is no stress in this member but a ½″ diameter rod will be used to prevent a possible sag in the lower chord.

Member LM. A steel rod will be used for this member since it resists a tensile force, the magnitude of which is 4,300#. In accordance with Art. 100 an initial stress of 1,000# will be assumed, therefore the force to be resisted will be 4,300 + 1,000 or 5,300#. We find by referring to Table XV that a steel rod ⅞″ in diameter will resist a safe load of 6,704# and therefore will be adequate for the member *LM*.

Member NO. A force of 17,200# is to be resisted by this tension member. Assume the initial stress to be 2,000#, making a total stress of 17,200 + 2,000 or 19,200#. Table XV shows that a 1½″ diameter steel rod will safely support this force and therefore is acceptable.

Since the truss and loading are symmetrical, it is necessary to design the members for one-half of the truss only; the two sides will be alike. The general construction of this truss is shown in Fig. 89. As explained in Art. 101, the working lines are first drawn and the members superimposed thereon, the center-of-gravity lines of the members coinciding with the working lines. Now that the sizes have been determined the remaining step is to detail the various joints. The type of joint is optional with the designer. However, it is customary to employ the simplest form of connection possible, with a minimum amount of cutting of material. The computations necessary in designing the connections are similar to those that have been previously described.

CHAPTER 12

STEEL TRUSSES

108. **Steel Trusses.** The use of structural steel shapes greatly facilitates the construction of roof trusses and, in localities in which it is readily available, steel is often found to be more economical than timber. The most common section employed consists of two angles having unequal legs with the long legs back to back, the connection at the joints being constructed by introducing a steel gusset plate between the angles. See Fig. 90. The angles are secured to the gussets by the use of rivets or welds.

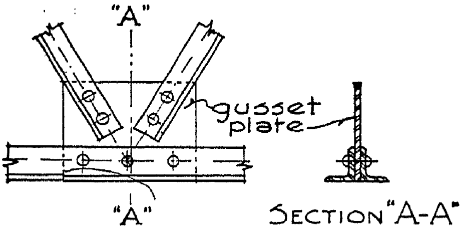

FIG. 90

Any type of truss constructed by the use of timber may be built of steel, but since the fabrication of steel trusses generally is less complicated, many types of steel roof trusses are impracticable if designed for timber.

Steel is an ideal material for resisting tensile forces. The Fink truss, Fig. 31, having a number of tension members and compression members of relatively short lengths, is an ideal type for steel construction. In addition, the strut web members frame into the upper chord at right angles, thus permitting simple framing details. Two other commonly used triangular trusses are the Howe and Pratt, the Pratt truss being particularly suited to steel construction and the Howe truss preferable when timber is to be employed. The type of truss to adopt depends entirely upon conditions, length of span, manner of loading, etc.

109. **Sizes of Sections.** To avoid eccentricity at the connections, it is advisable to employ two steel angles for truss members rather than single sections. These angles are placed back to back, the space between corresponding to the thickness of the gusset plate. Usually the angles have unequal legs and are placed with the short legs outstanding to provide greater stiffness. For larger trusses or for trusses in which the stresses are sufficiently great, two channel sections are sometimes used for the chord members. For trusses of average span, ¾″ diameter rivets are commonly used. The smallest leg of an angle in which a ¾″ rivet may be driven is 2½″ and the smallest angle commonly used for trusses is 2½″ x 2″ x ¼″. Some authorities require the minimum thickness of angles, when used as truss members, to be 5⁄16″.

Unless a member be subjected to transverse loads producing bending, such as an upper chord with purlins between panel points, truss members are simply struts or ties. A strut is a member resisting compression and if the force to be resisted is tension the member is called a tie.

110. **Steel Compression Members.** The allowable load that may be placed on a member in compression is determined by the use of a column formula. There are a number of such formulas and the designer will, of course, use the formula specified in the building code or regulations having jurisdiction. A column formula used by a great number of municipalities, and which may be used with safety is

$$f = \frac{18{,}000}{1 + \frac{1}{18{,}000}} \quad *$$

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* The 1936 specification of the American Institute of Steel Construction promulgates this formula for axially loaded columns with values of $\frac{l}{r}$ greater than 120. For axially loaded columns with values of $\frac{l}{r}$ not greater than 120, their formula is $f = 17{,}000 - 0.485\left(\frac{l}{r}\right)^2$. This specification gives 120 as the limit of $\frac{l}{r}$ for main compression members and 200 as the limit for bracing and other secondary members in compression.

in which

f = the allowable average unit stress, in pounds per square inch
l = the unsupported length of the column, in inches
r = the least radius of gyration with respect to the axis about which the column may bend, in inches

When using this formula, 15,000#/in^2 is the maximum permissible unit stress, the stress to be used for columns having a slenderness ratio, $\frac{l}{r}$, 60 or less. Likewise, $\frac{l}{r}$ must not exceed 120 for main compression members, and for bracing and other secondary members the ratio $\frac{l}{r}$ must not exceed 200.

To design a steel column, the design load and unsupported length of the column are first ascertained, the procedure then being as follows:

Step 1. Assume a trial section and determine, from the table of properties, the least radius of gyration and the area.

Step 2. Compute the slenderness ratio, $\frac{l}{r}$, both terms being in inches, and note that the ratio falls within 120, the prescribed limit.

Step 3. Compute f, the average allowable unit stress in the column formula, such as

$$f = \frac{18{,}000}{1 + \frac{}{18{,}000}}$$

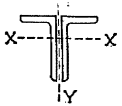
Step 4. Multiply the value of f found in Step 3 by the number of square inches in the section. This product is the allowable load *on the trial section that has been assumed.*

Step 5. The allowable load found in Step 4 is now compared with the design load, and if found to be less than the design load a larger cross section is selected and tested in a similar manner.

To illustrate the design of a steel column, let it be required to design a strut for a compression member in a truss, the length of the member being 10′0″ and the design load 76,000#. Following the above procedure,

Step 1. Assume the column section to consist of two 4″ x 3″ x ¼″ angles, separated by ⅜″ to provide for gusset plates, placed

Table XVII. Safe Loads for Struts of Two Angles,* Allowable Concentric Loads in Kips for Struts of Two Unequal Angles—3/8″ Back to Back, Long Legs Back to Back



Loads to right of heavy vertical lines are for secondary members only

Size	Thickness	Two angles		Radius of gyration		Axis X - X													Axis Y - Y												
						Unsupported length in feet													Unsupported length in feet												
		Area	Weight	X-X	Y-Y	4	5	6	7	8	9	10	12	14	16	18	20	22	4	5	6	7	8	9	10	12	14	16	18	20	22
2½ × 2	3/16	1.62	5.5	.79	.92	24	22	20	18	16	14	13	10	...	...	...	...	...	24	24	22	20	18	17	15	12	10				
	1/4	2.12	7.2	.78	.93	32	29	26	23	21	19	17	13	...	...	...	...	...	32	31	29	26	24	22	20	16	14				
	5/16	2.62	9.0	.78	.95	39	36	32	29	26	23	20	16	...	...	...	...	...	39	39	36	33	30	27	25	21	17				
3 × 2½	1/4	2.62	9.0	.95	1.13	39	39	36	33	30	28	25	20	17	...	...	...	...	39	39	38	36	34	31	29	25	21	18	16		
	5/16	3.24	11.2	.94	1.14	49	48	44	40	37	34	31	25	21	...	...	...	...	49	49	48	45	42	39	36	31	26	23	19		
	3/8	3.84	13.2	.93	1.16	58	56	52	48	43	40	36	30	25	...	...	...	...	58	58	57	53	50	47	43	37	32	27	24		
	7/16	4.42	15.2	.92	1.17	66	64	59	54	50	45	41	34	28	...	...	...	...	66	66	66	62	58	54	50	43	37	32	27		
3½ × 2½	1/4	2.88	9.8	1.12	1.09	43	43	42	39	37	34	32	27	23	20	17	...	...	43	43	42	39	36	34	31	26	22	19	16		
	5/16	3.56	12.2	1.11	1.10	53	53	52	49	45	42	39	33	28	24	21	...	...	53	53	52	48	45	42	39	33	28	24	20		
	3/8	4.22	14.4	1.10	1.11	63	63	61	57	53	50	46	39	33	28	24	...	...	63	63	62	58	54	50	46	39	33	29	24		
	7/16	4.86	16.6	1.09	1.12	73	73	70	66	61	57	52	44	38	32	26	...	...	73	73	71	67	62	58	53	46	39	33	29		
	1/2	5.50	18.8	1.09	1.13	83	83	80	75	69	64	59	50	43	36	31	...	...	83	83	81	76	71	66	61	52	44	38	33		
4 × 3	5/16	4.18	14.4	1.27	1.30	63	63	63	61	57	54	50	44	38	33	29	25	...	63	63	63	61	58	54	51	45	39	34	30	26	
	3/8	4.96	17.0	1.26	1.31	74	74	74	72	68	63	59	52	45	39	34	30	...	74	74	74	73	69	65	61	53	47	41	36	31	
	7/16	5.74	19.6	1,25	1.32	86	86	86	83	78	73	68	60	52	45	39	34	...	86	86	86	84	80	75	71	62	54	47	42	36	32
	1/2	6.50	22.2	1.25	1.33	98	98	98	94	88	83	77	67	58	51	44	38	...	98	98	98	96	91	86	81	71	62	54	47	42	37
	9/16	7.24	24.8	1.24	1.35	109	109	109	104	98	92	86	75	65	56	49	42	...	109	109	109	107	102	96	91	80	70	61	54	47	42
	5/8	7.96	27.2	1.23	1.36	119	119	119	114	107	100	94	81	70	61	53	46	...	119	119	119	118	112	106	100	88	78	68	60	52	46
4 × 3½	5/16	4.50	15.4	1.26	1.55	68	68	68	65	61	58	54	47	41	35	31	27	...	68	68	68	68	67	64	61	55	49	44	39	35	31
	3/8	5.34	18.2	1.25	1.56	80	80	80	77	72	68	64	55	48	42	36	32	...	80	80	80	80	79	76	72	65	58	52	47	41	37
	7/16	6.18	21.2	1.24	1.57	93	93	93	89	83	78	73	64	55	48	41	36	...	93	93	93	93	92	88	84	76	68	61	54	48	43
	1/2	7.00	23.8	1.23	1.58	105	105	105	100	94	88	82	72	62	54	46	40	...	105	105	105	105	105	100	95	86	77	69	62	55	49
	9/16	7.80	26.6	1.23	1.59	117	117	117	112	105	98	92	80	69	60	52	45	...	117	117	117	117	117	112	107	96	87	78	69	62	55
	5/8	8.60	29.4	1.22	1.60	129	129	129	123	115	108	101	87	75	65	56	49	...	129	129	129	129	129	123	118	107	96	86	77	69	62
	11/16	9.36	32.0	1.21	1.62	140	140	140	133	125	117	109	94	81	70	61	53	...	140	140	140	140	140	135	129	117	105	95	85	76	68
	3/4	10.12	34.6	1.20	1.63	152	152	152	143	134	126	117	101	87	75	65	57	...	152	152	152	152	152	146	140	127	115	103	92	83	74
5 × 3	5/16	4.80	16.4	1.61	1.22	72	72	72	72	72	69	66	60	54	48	43	39	35	72	72	72	68	64	60	56	49	42	36	32	27	
	3/8	5.72	19.6	1.61	1.23	86	86	86	86	86	82	79	71	64	57	51	46	41	86	86	86	82	77	72	67	59	51	44	38	33	
	7/16	6.62	22.6	1.60	1.24	99	99	99	99	99	95	91	82	74	66	59	53	47	99	99	99	95	89	84	78	68	59	51	44	39	
	1/2	7.50	25.6	1.59	1.25	113	113	113	113	112	107	103	93	83	75	67	60	53	113	113	113	108	102	95	89	78	67	58	51	44	
	9/16	8.36	28.6	1.58	1.26	125	125	125	125	125	119	114	103	92	83	74	66	59	125	125	125	121	114	107	100	87	76	66	57	50	
	5/8	9.22	31.4	1.57	1.28	138	138	138	138	137	131	125	113	101	91	81	72	64	138	138	138	134	126	119	112	98	85	74	64	56	
	11/16	10.06	34.2	1.56	1.29	151	151	151	151	150	143	136	123	110	98	88	78	70	151	151	151	147	138	130	122	107	93	81	71	62	
	3/4	10.88	37.0	1.55	1.31	163	163	163	163	161	154	147	132	118	106	94	84	75	163	163	163	159	151	142	134	117	102	89	78	68	
5 × 3½	5/16	5.12	17.4	1.61	1.45	77	77	77	77	77	74	70	64	57	51	46	41	37	77	77	77	77	74	71	67	60	53	47	41	37	
	3/8	6.10	20.8	1.60	1.46	92	92	92	92	92	88	84	76	68	61	55	49	44	92	92	92	92	89	84	80	71	63	56	50	44	
	7/16	7.06	24.0	1.59	1.47	106	106	106	106	106	101	97	87	78	70	63	56	50	106	106	106	106	103	98	93	83	74	65	58	51	
	1/2	8.00	27.2	1.58	1.49	120	120	120	120	120	114	109	99	88	79	71	63	56	120	120	120	120	117	112	106	95	84	75	66	59	
	9/16	8.94	30.4	1.57	1.50	134	134	134	134	133	127	121	110	98	88	78	70	62	134	134	134	134	131	125	119	107	95	84	75	66	
	5/8	9.84	33.6	1.56	1.51	148	148	148	148	146	140	133	120	108	96	86	76	68	148	148	148	148	145	138	131	118	105	93	83	74	
	11/16	10.74	36.6	1.56	1.52	161	161	161	161	160	153	146	131	118	105	94	83	75	161	161	161	161	158	151	144	129	115	103	91	81	
	3/4	11.62	39.6	1.55	1.54	174	174	174	174	172	165	157	141	127	113	101	90	80	174	174	174	174	172	164	156	141	126	112	100	89	

---

* Loads are computed by formula $f = \dfrac{18{,}000}{1 + \dfrac{1}{18{,}000}\left(\dfrac{l}{r}\right)^2}$

with the long legs back to back. Referring to Table XVII note that the least radius of gyration is 1.25″ and the area of the section is 6.5in^2.

Step 2. The slenderness ratio, $\frac{l}{r} = \frac{10 \times 12}{1.25} = 96$. Since the limiting ratio is 120, the section is acceptable with respect to this requirement.

Step 3. Using the column formula

$$f = \frac{18{,}000}{1 + \frac{1}{\phantom{18{,}000}}\left(\frac{l}{r}\right)^2} \quad \text{or} \quad f = \frac{18{,}000}{1 + \frac{1}{18{,}000}(96)^2}$$

or

$$f = 11{,}905\#/\text{in}^2$$

Step 4. The area of the cross section multiplied by the average allowable unit stress is 6.5 × 11,905 = 77,382#, the allowable load on the column.

Step 5. Since the design load is 76,000# and the permissible load on the column is 77,382#, the assumed column section is acceptable.

The above computations illustrate the method of applying the column formula in designing compression members. In practice, however, the proper sections may be selected directly by referring to a safe-load table such as Table XVII. Note that this table gives an allowable load of 77 kips for a section composed of two 4″ x 3″ x ½″ angles having a length of 10′0″, the same result found by computations.

In using Table XVII *notice the heavy vertical lines.* The loads to the right of these lines are for columns having a slenderness ratio greater than 120. This table is reproduced by permission of the American Institute of Steel Construction as published in the 1930 edition of their Manual of Steel Construction.

For the above example we find, by referring to the table, that two radii of gyration are given, one with respect to the X-X axis and the other with respect to the axis Y-Y, 1.25″ and 1.33″ respectively. The allowable load is always determined in accordance with the *least* radius of gyration.

111. Steel Tension Members. If a member in tension fails, the section will tear apart at the point where the area of the cross section is the smallest. This reduced area occurs at a rivet hole. Rivets are generally placed in a single line and therefore the net section is the gross section of the angle minus the area resulting from multiplying the diameter of the hole by the thickness of the angle. See Fig. 91 (*b*). For angles having legs 5″ and greater, the rivets are placed in two lines but are staggered as indicated in the lower chord of the joint in Fig. 91 (*a*). The holes for rivets are punched 1/16″ greater than the diameter of the rivet but, in computing the area to be deducted, the diameter is assumed to be 1/8″ larger than the diameter of the rivet. The additional 1/16″ is to compensate for damage to the material which results from punching.

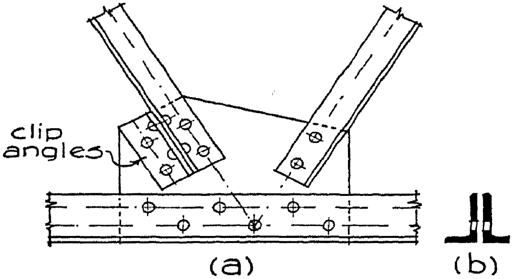


FIG. 91

To illustrate the method of computing the allowable tensile load on member, consider a 5″ x 3½″ x ½″ angle punched for a ¾″ rivet. The area to be deducted for the rivet hole is ½ × 7/8 or 7/16in². The gross area of the angle is 4in², Table XVIII, therefore the net area equals (4 − 7/16) or 3 9/16in². The allowable tensile unit stress of steel* is generally given as 18,000#/in², hence the allowable tensile load on the net section of the angle is 3 9/16 × 18,000 or 64,125#.

Suppose 7/8″ rivets are used in place of the ¾″ rivets. Then the net area equals 4 − (1 × ½) = 3½in². Similarly, the allowable tensile load will be 3½ × 18,000 or 63,000#.

* The allowable tensile unit stress of structural steel given in the 1936 American Institute of Steel Construction specification is 20,000#/in².

As in designing members in compression, members resisting tensile forces are most readily designed by making use of safe-load tables such as Table XVIII. In using this table note that the allowable tensile unit stress is 18,000#/in^2 and that the allowable loads are for *single* angles with *one* rivet hole deducted. Theoretically, angles should be connected by rivets in both legs to develop their full tensile strength. In large members the use of clip angles permits rivets in both legs, and for smaller members a slight increase in size of the angle provides an adequate allowance.

Table XVIII. Tension Values of One Angle with One Rivet Hole Deducted

Allowable Values in Units of 1,000#

Size, In	Gross area, In^2	Allowable net stress @ 18,000#/in^2	
		7/8″ rivets	3/4″ rivets
2½ x 2 x 3/16	0.81		11.5
1/4	1.06		15.1
5/16	1.31		18.7
3 x 2½ x 1/4	1.31	19.1	19.6
5/16	1.62	23.6	24.3
3/8	1.92	27.7	28.6
7/16	2.21	31.9	32.8
3½ x 2½ x 1/4	1.44	21.4	22.0
5/16	1.78	26.5	27.2
3/8	2.11	31.1	32.0
7/16	2.43	35.8	36.7
1/2	2.75	40.5	41.6
4 x 3 x 5/16	2.09	32.0	32.8
3/8	2.48	37.8	38.7
7/16	2.87	43.7	44.6
1/2	3.25	49.5	50.6
9/16	3.62	55.1	56.3
5/8	3.98	60.3	61.7
4 x 3½ x 5/16	2.25	34.9	35.6
3/8	2.67	41.2	42.1
7/16	3.09	47.7	48.6
1/2	3.50	54.0	55.1
5/8	4.30	66.1	67.5
11/16	4.68	71.8	73.4
3/4	5.06	77.6	79.2
5 x 3 x 5/16	2.40	37.6	38.3
3/8	2.86	44.6	45.5
7/16	3.31	51.7	52.6
1/2	3.75	58.5	59.6
9/16	4.18	65.2	66.4
5/8	4.61	71.6	73.1
11/16	5.03	78.1	79.7
3/4	5.44	84.4	86.0
5 x 3½ x 5/16	2.56	40.5	41.2
3/8	3.05	48.1	49.0
7/16	3.53	55.6	56.5
1/2	4.00	63.0	64.1
9/16	4.47	70.4	71.6
5/8	4.92	77.2	78.7
11/16	5.37	84.2	85.9
3/4	5.81	91.1	92.7
6 x 3½ x 3/8	3.42	54.7	55.6
7/16	3.97	63.5	64.4
1/2	4.50	72.0	73.1
9/16	5.03	80.5	81.7
5/8	5.55	88.6	90.0
11/16	6.06	96.7	98.3
3/4	6.56	104.6	106.2
13/16	7.06	112.5	114.3
7/8	7.55	120.1	122.0

Let it be required to determine the size of a member of a truss to resist a tensile force of 60,000# using $\frac{3}{4}''$ rivets. Since truss members invariably consist of two angles, each individual angle will be required to resist a tensile force of 60,000 ÷ 2, or 30,000#. Referring to Table XVIII we find that a $4'' \times 3'' \times \frac{5}{16}''$ angle, punched for a $\frac{3}{4}''$ rivet, has an allowable tensile load of 32,800#, consequently a member consisting of two $4'' \times 3'' \times \frac{5}{16}''$ angles is acceptable.

112. **Working Values of Rivets.** Although welding is used to a considerable degree, the most common method employed to form the connections for truss members is by the use of rivets.

A rivet may fail by shearing of the shank as indicated by Fig. 92 (*a*) or by bearing, Fig. 92 (*b*). By bearing is meant the tendency of the cylindrical surface to cut into the adjacent plate or leg of an angle.

When two members are held together by a rivet as shown in Fig. 92 (*a*), there is a tendency for the two pieces of metal to slide one on the other. This tendency is resisted by the rivet. If the rivet fails by shearing, it will shear in a plane indicated by the dotted line. This rivet is said to be in *single shear*. The allowable working value of one rivet in single shear is the area of the shank of the rivet multiplied by the allowable shearing stress of rivet steel. For power-driven rivets the allowable shearing stress is frequently given as 13,500#/in^2.

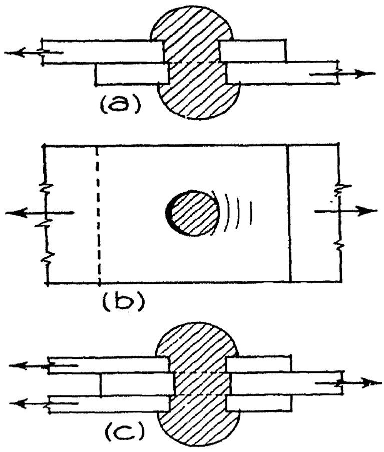


FIG. 92

When two angles transfer their stress to a gusset plate, the gusset is placed between the legs of the angles and the condition indicated in Fig. 92 (*c*) results. For the rivet to fail by shearing, two surfaces must fail as shown by the dotted lines. This rivet is said to be in *double shear*. The allowable working value of a rivet in double shear is two times its value for single shear, there being two planes along which the rivet tends to fail by shearing.

In addition to shearing of the rivet, there is also a tendency for the connection to fail by crushing of the plate where the bearing pressure is exerted by the rivet. In Fig. 92 (*a*) the rivet is said to be in *single bearing* since the rivet is in single shear. The area assumed to resist bearing is a rectangle the dimensions of which are the thickness of the plate or angle and the diameter of the rivet. Hence, the allowable working value of a rivet in bearing is this area multiplied by the allowable bearing unit stress. When a rivet is in single shear, the plates or legs of the angles are in single bearing. The allowable bearing unit stress for single bearing is commonly given as 24,000#/in^2.

Since, in trusses, the members generally consist of two angles separated by gusset plates at their connections, a condition of double shear exists as shown in Fig. 92 (*c*). The two outer members are in single bearing but the enclosed plate is said to be in *double bearing*. For the enclosed plate, the stresses transferred by the rivet are more uniformly distributed and in addition there is a degree of friction between the plates. Owing to this, the allowable bearing unit stress of a rivet on a plate tightly enclosed between two others (double bearing) is somewhat greater than for a rivet and plate in single bearing, the allowable stress being 30,000#/in^2.

The allowable unit stresses just given, namely,

Shearing unit stress	13,500#/in^2
Single-bearing unit stress	24,000#/in^2
Double-bearing unit stress	30,000#/in^2

are stresses given for power-driven rivets in a great number of building codes and may be used with safety.*

To determine the number of rivets to use in order to transfer the stress from a member to a gusset plate, the thickness of the angles, the thickness of the gusset plate and the diameter of the rivet are first ascertained. The next step is to find the controlling working value of one rivet. This value being determined, it is divided into the load on the member and the required number of rivets thus obtained.

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* The 1936 specification of the American Institute of Steel Construction gives the following allowable unit stresses for rivets: shearing unit stress, 15,000#/in^2; single-bearing unit stress, 32,000#/in^2; double-bearing unit stress, 40,000#/in^2.

To find the controlling value of a rivet, consider the following data. A 3/8″ gusset plate occurs between two angles, the thickness of the legs being 1/4″. The diameter of the rivet is 7/8″. This condition is indicated in Fig. 92 (*c*). Note that the rivet is in double shear, the gusset plate is in double bearing and the legs of the angles are in single bearing since they are not enclosed.

Double Shear. The area of the rivet is $\frac{\pi d^2}{4}$ or $\frac{3.1416 \times 0.875^2}{4} =$ 0.6013in^2. Since the allowable shearing unit stress is $13{,}500\#/\text{in}^2$, $0.6013 \times 13{,}500 = 8{,}117\#$, the allowable working value of one 7/8″ rivet in single shear. Since, however, the rivet in this instance is in double shear, $2 \times 8{,}117 = 16{,}234\#$, the allowable working value of one 7/8″ rivet in double shear.

Double Bearing. The enclosed 3/8″ gusset plate is in double bearing. The assumed area of the rivet in bearing against the plate is the diameter of the rivet multiplied by the thickness of the gusset. Hence the area in bearing is 0.875×0.375 or 0.3281in^2. This area multiplied by the allowable unit stress for double bearing is $0.3281 \times 30{,}000$ or 9,843#, the allowable working value of a 7/8″ rivet on a 3/8″ gusset plate in double bearing.

Single Bearing. Considering first only one of the angle legs, the bearing area is 0.875×0.25 or 0.21875in^2. Since this angle leg is in single bearing, the allowable bearing unit stress is $24{,}000\#/\text{in}^2$, and hence the working value is $0.21875 \times 24{,}000$ or 5,250#. There being two angle legs, one on each side of the gusset, the allowable working value of the 7/8″ rivet on the two angle legs in single bearing is $2 \times 5{,}250$ or 10,500#.

Thus we have found, by computations, the working values to be:

7/8″ rivet, double shear = 16,234#
3/8″ gusset, double bearing = 9,843#
Two 1/4″ thick angles, single bearing,
  2 × 5,250 = 10,500#

Of these three values, 9,843# is the smallest and therefore is *the controlling value*. To find the number of rivets required for a member, we merely divide the controlling value of one rivet into the load on the member.

Two is the smallest number of rivets used for any connection.

At a joint in the upper chord at which a purlin produces a force, a sufficient number of rivets should be used in the connection to resist the reactions of the purlins as well as the forces in the chord members.

113. **Working Values of Rivets by Use of Tables.** It is not to be supposed that working values for rivets need be computed for the usual connections. These values may be found directly by the use of Table XIX. Note that the unit stresses used in compiling this table are similar to those given in Art. 112. In this table bearing values are not given beyond the point where shear gov-

Table XIX. Allowable Working Values for Power-Driven Rivets

Unit Shearing Stress = 13,500#/in^2
Unit Bearing Stress { Single Bearing = 24,000#/in^2
Double Bearing = 30,000#/in^2

Diameter of rivet		⅝ in		¾ in		⅞ in		1 in		1⅛ in	
Area of rivet		0.3068		0.4418		0.6013		0.7854		0.9940	
Single shear pounds		4,140		5,960		8,120		10,600		13,420	
Double shear pounds		8,280		11,930		16,240		21,200		26,840	
Bearing		Single Lb	Double Lb	Single Lb	Double Lb	Single Lb	Double Lb	Single Lb	Double Lb	Single Lb	Double Lb
Thickness of plate in inches	¼	3,750	4,690	4,500	5,630	5,250	6,560	6,000	7,500	6,750	8,440
	5/16	....	5,860	5,630	7,030	6,560	8,200	7,500	9,380	8,440	10,550
	⅜	....	7,030	....	8,440	7,880	9,840	9,000	11,250	10,130	12,660
	7/16	....	8,200	....	9,840	....	11,480	10,500	13,130	11,810	14,770
	½	....	.....	....	11,250	....	13,130	.....	15,000	.....	16,880
	9/16	....	.....	....	.....	....	14,770	.....	16,880	.....	18,980
	⅝	....	.....	....	.....	....	.....	.....	18,750	.....	21,090

erns. By use of Table XIX verify the allowable working values found in the previous illustration.

Let it be required to compute the number of rivets in the upper and lower chords at the joint as indicated in Fig. 93 (*a*). Note the size of the members and their stresses. It will be assumed that ¾″ rivets are to be used and that the gusset plate is ⅜″ in thickness.

Considering first the upper chord member, the thickness of the angle legs is 5/16″, the gusset is ⅜″ and the rivet is ¾″ in diameter.

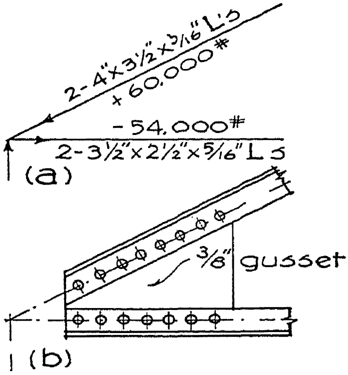


FIG. 93

The rivet and angle legs are similar to the condition represented in Fig. 92 (*c*). Referring to Table XIX the following allowable working values are found:

¾″ rivet, double shear = 11,930#
⅜″ gusset, double bearing = 8,440#
controlling value
Two 5/16″ angle legs, single bearing,
2 × 5,630 = 11,260#

Of these values, 8,440# is the smallest and therefore is the controlling value. The load on the upper chord is 60,000#, hence 60,000 ÷ 8,440 = 7+. Eight rivets will be required in this member.

As the rivets, the thickness of the angles and gusset for the lower chord connection are similar to those of the upper chord, the same controlling value, 8,440#, will apply. The stress in the lower chord being 54,000#, 54,000 ÷ 8,440 = 6+; hence seven rivets will be required.

114. **Rivets at Joints of Continuous Members.** If the length of the upper chord of a truss is not too great, it is customary to use one continuous member, the cross section of which is determined by the greatest stress in any member of the chord. The apparent waste of material in using a continuous member is offset by the saving in cost of fabrication.

The joint illustrated in Fig. 94 (*b*) shows the upper chord to consist of two 3″ x 2½″ x $\frac{5}{16}$″ angles. Assuming the gusset plate to be ⅜″ in thickness and the rivets to be ¾″ in diameter, the controlling value of one rivet is 8,440# as explained in Art. 113. If the member of the upper chord, having a stress of 32,000# as indicated in Fig. 94 (*a*), *were not continuous* at this joint, the number of rivets required in the member would be 32,000 ÷ 8,440 or 3+, necessitating four rivets. Similarly, three rivets would be required for the member having a stress of 24,000#.

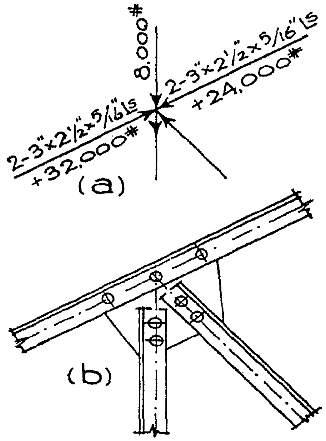


FIG. 94

In this instance, however, *the upper chord is continuous* and hence the rivets are not required to transmit the entire chord stresses on either side of the joint. *The load transmitted by the rivets for such a condition is the difference in magnitude of the two stresses.* Thus

32,000# − 24,000# = 8,000#, and the number of rivets required is 8,000 ÷ 8,440 or, theoretically, less than one. As has been stated, two is the minimum number of rivets used at any connection. Rivets should never have a pitch (spacing) exceeding 6″ and enough rivets should always be used to fill out the gusset, placing a rivet near the edge of the gusset to provide stiffness. Therefore, because of the size of the gusset, three rivets are used in the upper chord at this joint. In Fig. 94 (*a*) note that the load from the purlins is 8,000#. There should always be a sufficient number of rivets in the joint to develop the load from the purlins. In this instance the three rivets are ample. The number of rivets in the chords of a truss should always be at least equal to the greatest number in any member framing into the joint.

115. **Gusset Plates.** The function of a gusset plate is to assist in transferring stresses from one member to another. Its area is determined by the required number of rivets in the members. In general, a gusset is made as small as possible, thus providing a minimum weight. The most common thickness employed for trusses of average span is ⅜″. For web members, where the stresses are relatively small, a 5⁄16″ gusset is sometimes used. At the heel and peak joints a gusset ½″ in thickness may be used to reduce the number of rivets.

When ¾″ rivets are used with ⅜″ gussets, the controlling value of one rivet is 8,440#, the value for double bearing on the plate. If the gusset is increased to ½″ thickness, this value becomes 11,250#, thus permitting a smaller number of rivets and consequently a smaller area of gusset plate. If, in this case, the gusset is further increased to ⅝″, the value in double bearing becomes 14,062#. But the ¾″ rivet has a value of only 11,930# in double shear, hence it is of no advantage to use a gusset whose value in double bearing exceeds the value of the rivet in double shear.

Rectangular gusset plates are used when possible, but the position of the truss members and the required number of rivets frequently make this impracticable. Theoretically, the lines of action of the truss members should coincide with the lines passing through the centroids of the sections. For the smaller angles, the gage lines of the rivets are superimposed on the working lines. The arrangement of gusset and rivets should be such that eccentricity is avoided. As an example, the joint shown in Fig. 95 (*a*) would produce eccentric loading in the rivets and therefore the joint shown in Fig. 95 (*b*) is used.

A method often used to reduce the size of the gusset plate is the use of *clip angles* as indicated in Fig. 91. Since 6″ is considered to be the maximum spacing of rivets, it frequently happens that the size of a gusset determines the number of rivets to be used. This is illustrated in the lower chord in Fig. 91.

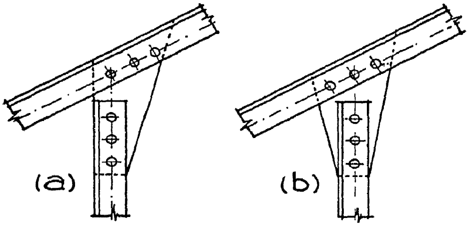


FIG. 95

116. Gage Lines. To make a drawing showing the arrangement of the members, gussets, rivets, etc., the first step is to lay out the *working lines*. This system of lines corresponds to the truss diagram from which the stress diagram is constructed. The size of

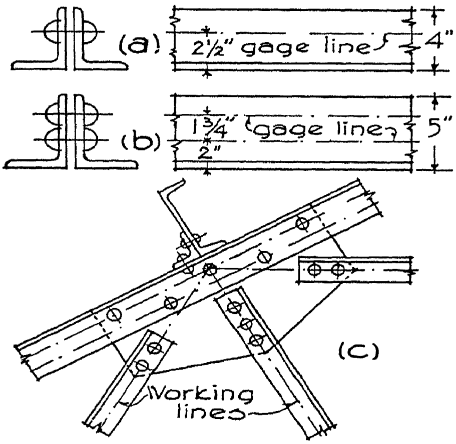
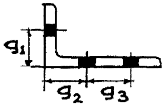


FIG. 96

the members having been determined, they are superimposed on the gage lines as indicated in Fig. 96 (*c*). In theory, the working lines and center-of-gravity lines should coincide, but practical considerations make this impracticable. Rivets cannot be driven closer to the outstanding leg of an angle than the minimum distance required for the riveting tool. *Gage lines* are the lines parallel to the length of a member on which the rivets are placed. Gage dimensions for standard angles are shown in Table XX. For angles having legs 4″ and less, the rivets are driven in a single line and for larger angles two rows of rivets are used, that is, there are two gage lines. See Fig. 96 (*a*) and (*b*). *The gage lines are made to coincide with the working lines.* When the angle has two gage lines, the gage line nearest the back of the angle is made coincident with the working line as shown in Fig. 96 (*c*). The gage lines intersect at a common point at the joints, and the rivets are driven on these lines.

Table XX. Gage Dimensions for Standard Angles



Leg	2	2½	3	3½	4	5	6	7	8
g_1	1⅛	1⅜	1¾	2	2½	3	3½	4	4½
g_2	. . .	. . .	. . .	. . .	. . .	2	2¼	2½	3
g_3	. . .	. . .	. . .	. . .	. . .	1¾	2½	3	3
Maximum rivet. . .	⅝″	¾″	⅞″	⅞″	⅞″	⅞″	⅞″	1″	1⅛″

117. **Rivet Dimensions.** For the most common trusses, ¾″ or ⅞″ diameter rivets are customary. In the smaller trusses ¾″ rivets are invariably used. Regardless of the magnitude of the stress, two is the minimum number of rivets to use in each member at a connection.

The distance center to center, in a direction parallel to the length of a member, is called the *pitch* of the rivets whether they are on the same or different gage lines. The minimum pitch is three diameters of a rivet, 2½″ being the usual standard minimum and 6″ the maximum. The usual minimum *edge distance* is 1¼″. When the required number of rivets exceeds seven in a single-gage angle, a *clip angle* is generally used as shown in Fig. 91.

Between joints the two angles forming the members should be riveted, with washers inserted between the backs of the angles, to permit the two angles to act in unison. These are called *stitch rivets* and should be spaced uniformly. Obviously, they are an absolute necessity for members in compression but they are used in tie members as well. In practice, 2′0″ is the maximum spacing for stitch rivets in compression members and for tension members 3′6″ is generally the maximum.

118. **Steel Purlins.** The steel sections generally used for purlins are I-beams and channels. For pitched roofs it is customary to use *sag rods* to provide stiffness in the direction parallel to the roof surface. In addition to this, the sag rods keep the purlins in line during erection of the roofing materials. Depending on the size of the purlin, sag rods are usually ⅝″ or ¾″ in diameter and are placed at the center of the purlin span, 3″ below the top flange of the purlin as shown in Fig. 97 (*a*). The purlin is designed as a simple beam with a uniformly distributed load equal to the normal component of the vertical load. The component of the vertical load, parallel to the roof surface, is resisted by the sag rods. At the ridge of the roof two purlins may be used as shown in Fig. 104, or a single purlin framed to the extended gusset may be employed as illustrated in Fig. 97 (*d*). In any event, the sag rods should ex-

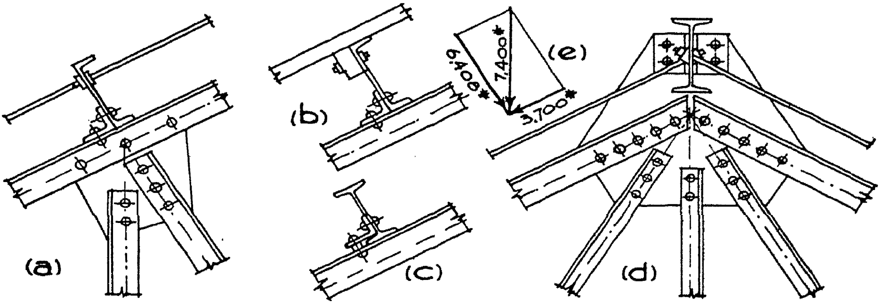


Fig. 97

tend from eave to eave in a continuous line over the ridge of the roof. Frequently it is necessary to increase the size of the ridge purlins to provide for the vertical component of the stresses in the sag rods.

To illustrate the design of a purlin, assume the uniformly distributed vertical load to be 7,400#, including the weight of the purlin, and the length of the purlin between trusses to be 17′0″. The roof has a 30° pitch. A force polygon is drawn, Fig. 97 (*e*), and the two components, one normal and the other parallel to the roof, are found to be 6,408# and 3,700# respectively. The sag rod will resist the latter force, hence the purlin will be required to resist a load of 6,408# normal to the roof surface, the beam bending about an axis perpendicular to its web. This being a simple beam having a uniformly distributed load of 6,408# and a span of 17′0″, the maximum bending moment is

$$M = \frac{Wl}{8} \text{ or } M = \frac{6{,}408 \times 17 \times 12}{8} = 163{,}404''\#$$

$$\frac{M}{f} = S$$

Hence

$$\frac{163{,}404}{18{,}000} = 9\text{in}^3 \text{ the required section modulus.*}$$

Referring to a table giving the properties of standard channels, we find that a 9″ 13.4# channel has a section modulus of 10.5in^3 and therefore is adequate with respect to bending. However, this section will produce excessive deflection, consequently a 10″ 15.3# channel is accepted.

Since the sag rods will be placed at the center of the purlin between trusses, the tensile force to be resisted by one sag rod is 3,700 ÷ 2 or 1,850#. The theoretical area required for the rod will be 1,850 ÷ 16,000 or 0.115in^2. A rod ⅝″ in diameter provides a net area of 0.202in^2, Table XV, and will be acceptable.

119. **Sub-Purlins.** Small steel sections, frequently T-bars, running at right angles to the main purlins are known as *sub-purlins*,

---

* In this illustration an allowable extreme fiber stress of 18,000#/in^2 is used. This unit stress is given in a great number of building codes. The 1936 specification of the American Institute of Steel Construction gives this stress as 20,000#/in^2.

Fig. 98. Between these members are placed *book-tile* or *precast slabs* of gypsum or light-weight concrete. Over these tiles or slabs is laid a 2″ layer of nailing concrete as a base for the roofing material. Ribbed steel plates laid over the sub-purlins are frequently used for flat roofs as a foundation for built-up roofing. A concrete slab extending from purlin to purlin results in a comparatively heavy roof and presents difficulties in construction for steep pitches, hence the use of sub-purlins and precast slabs is often found to be advantageous. In any type of roof construction adequate insulation should be provided to overcome heat losses and the presence of water of condensation.

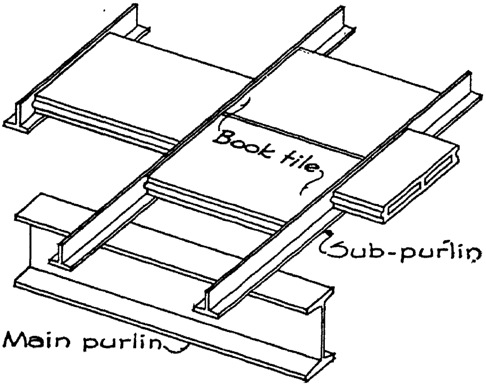


Fig. 98

120. **Members in Combined Compression and Bending.** The most common type of truss loading consists in purlins framing into the upper chord at the panel points. Occasionally, however, the type of roof construction demands that purlins occur between the panel points as illustrated in Fig. 99 (*a*). This results in the upper chord having combined stresses of compression and bending, that is to say, the chord must serve as a beam as well as a strut.

The design of such a member is accomplished by the trial method. A specific section is first assumed and by means of a column formula the maximum allowable unit stress is determined. Next, the maximum actual stress in the assumed section is computed by the use of the formula $f = \frac{P}{A} + \frac{Mc}{I}$, in which $\frac{P}{A}$ is the unit stress due to the compressive axial load, and $\frac{Mc}{I}$ is the maximum unit stress resulting from bending due to the intermediate purlin load.* The actual stress compared with the stress permitted by the column formula indicates whether or not the trial section is sufficiently large.

As an example, let it be required to design the upper chord member adjacent to the heel joint of the truss shown in Fig. 99 (*a*) and (*b*). The axial compressive load on the member is 42,000# and the load from the intermediate purlin causing bending is 3,200#. The length of the entire upper chord is 24′0″, hence the length of the member between panel points is 24 ÷ 3 or 8′0″.

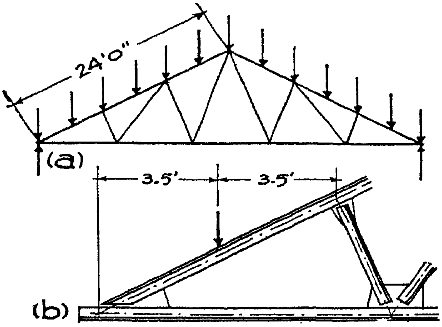


FIG. 99

Assume the cross section of the member to consist of two 5″ x $3\frac{1}{2}$″ x $\frac{1}{2}$″ angles with the long legs back to back separated by $\frac{3}{8}$″ for the gusset plate. Referring to a table giving the properties of angles† and Table XVII, the following data is noted. For the assumed trial section, the area of the two angles $A = 8\text{in}^2$; the weight per linear foot = 27.2#; the moment of inertia $I_{x-x} = 2 \times 10 = 20\text{in}^4$; the distance of the most remote fiber from the neutral axis $c = 3.33''$; and the least radius of gyration $r_{y-y} = 1.49''$.

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[*] $f = \frac{Mc}{I}$ is the flexure formula. See Art. 39, *Simplified Engineering for Architects and Builders.*

[†] See any steel handbook or Table XI, *Simplified Engineering for Architects and Builders.*

The slenderness ratio, $\frac{l}{r} = \frac{8 \times 12}{1.49} = 64.4$, acceptable since it is within the limit 120. See Art. 110.

The formula for steel columns is

$$f = \qquad\qquad \text{(See Art. 110.)}$$

$$1 + 18{,}000 \setminus r$$

Substituting,

$$\frac{18{,}000}{1 + \frac{1}{18{,}000}(64.4)^2} \qquad 14{,}629\#/\text{in}^2$$

This is the maximum allowable unit compressive stress for the assumed trial section.

The average compressive stress due to the direct compressive load of 42,000# is $\frac{P}{A}$ or $42{,}000 \div 8 = 5{,}250\#/\text{in}^2$. In this equation eight is the number of square inches in the cross section.

The bending in the member is produced by the load, 3,200#, from the intermediate purlin and also the weight of the member, 27.2 pounds per linear foot. The upper chord will be one continuous section, hence for bending it must be considered as a continuous beam. First, however, assume the member to be a simple beam between the heel and first panel point, then for the concentrated or purlin load,

$$M = Pl \_ \; 3{,}200 \times 7 \times 12 \qquad 67{,}200''\#$$

Note in Fig. 99 (*b*) the span length used in computing the bending moment is the *horizontal* distance between supports, 7′0″, not the length of the member.

For the uniformly distributed load due to the weight of the member, the value of the bending moment is

$$M = \frac{Wl}{\ } \text{ or } M = \frac{27.2 \times 8 \times 7 \times 12}{\ } \text{ or } M \qquad 2{,}285''\#$$

The maximum bending moment due to both concentrated and uniformly distributed loads is 67,200 + 2,285 or 69,485″#.

However, since this is a continuous beam, the bending moment is considered to be only 0.8 times the value of the bending moment of a beam simply supported, or 69,485 × 0.8 = 55,588″#.

Now returning to the fundamental formula for combined bending and compressive stresses,

and substituting the known values,

$$f = 5{,}250 -$$

This is the actual maximum unit compressive stress and occurs at the bottom fiber of the section over the first panel point. This stress being in reasonable agreement with the stress 14,629#/in^2 permitted by the column formula, the assumed trial section is acceptable.

121. **Members in Combined Tension and Bending.** When a ceiling is suspended from a truss, two general methods of framing are possible. First, beams may be run from truss to truss at the panel points of the lower chord in the manner that purlins are framed to the truss joints of the upper chord. In this instance the lower chord members are in direct tension. The second method is to have beams extend from truss to truss framing to the lower chord at regular intervals between joints of the truss, thus producing bending stresses as well as direct tension. In this case, the lower chord must serve as a beam in addition to resisting axial tension stresses.

In actual construction, the lower chord is invariably made continuous for two or more panel lengths and, therefore, in computing the bending moment, the member must be considered as a continuous beam. It is sufficiently accurate to consider both the positive and negative bending moments for a continuous beam to be 0.8 times the bending moment of a simple beam similarly loaded.

The principle involved in the design of a member in combined bending and direct tension is to assume a trial section and compute f, the maximum tensile unit stress in the formula $f = \frac{P}{A} + \frac{Mc}{I}$. In this formula $\frac{P}{A}$ is the direct axial tensile unit stress, and $\frac{Mc}{I}$ is the maximum tensile unit stress due to bending resulting from the loads between the joints. The value of f, thus found, is compared with 18,000#/in², the usual allowable tensile unit stress of structural steel, to see whether or not the trial section is sufficiently large.

To illustrate the procedure, let it be required to design that portion of the lower chord of a truss adjacent to the support, the member being continuous over the first interior truss joint. The member is to resist a direct axial tensile stress of 30,000# and, in addition, a uniformly distributed load of 500 pounds per linear foot resulting from a suspended ceiling and the weight of the member. The length of the member is 10′0″ and the allowable tensile unit stress is 18,000#/in².

Assume a trial section consisting of two 5″ x 3½″ x ⅜″ angles, ⅜″ apart, with long legs back to back, having the outstanding legs at the bottom.

Referring to a table of properties of standard angles* as well as to Table XVII, the following data is found. For the two angles the area $A = 6.1\text{in}^2$; the moment of inertia $I = 2 \times 7.8 = 15.6\text{in}^4$; the distance of the most remote fiber from the neutral surface $c = 3.39''$.

The direct axial tensile load is 30,000# and since the area of the section is 6.1in², the direct axial unit tensile stress is $\frac{P}{A}$ or $\frac{30,000}{6.1} = 4,918\#/\text{in}^2$.

Assuming first that the member is a simple beam, the maximum bending moment $M = \frac{Wl}{8}$, in which W is the total uniformly distributed load and l is the length of the member. Thus $M = \frac{Wl}{8}$ or $M \quad 500 \times 10 \times 10 \times 12 = 75,000''\#$. However, since the beam is continuous, the value of the maximum bending moment is considered to be $0.8 \times 75,000$ or $60,000''\#$.

Then $f = \frac{P}{A} + \frac{Mc}{I}$, and substituting the known values,

$$4,918$$

---

* See Table XI, *Simplified Engineering for Architects and Builders.*

the actual maximum unit tensile stress. Since the member is a continuous beam, this stress occurs at the top of the section at the first interior panel point. The actual unit stress being less than 18,000#/in^2, the allowable, the section is acceptable. If the actual unit stress had exceeded the allowable, a larger trial section would have been assumed and the process repeated.

122. **Heel Joints.** The heel joint is the name given to the joint of a triangular truss where the upper and lower chords meet at the support. In general, trusses are supported by masonry or by columns. Typical joints for trusses resting on masonry walls are shown in Fig. 100 (*a*), (*b*), (*c*) and (*d*), and truss connections to H-columns are illustrated in Fig. 100 (*e*) and (*f*). A typical form of connection, as shown in Fig. 100 (*b*) and (*c*), consists of *shoe angles* riveted to the gusset plate to permit the center line of the support to meet the intersection of the working lines of the upper and lower chords. The rivets in the angles over the bearing should be of sufficient number to transfer the end reaction of the truss to the bearing plate. In addition, the net section of the gusset plate should be large enough to resist the vertical shear which is equal in magnitude to the reaction.

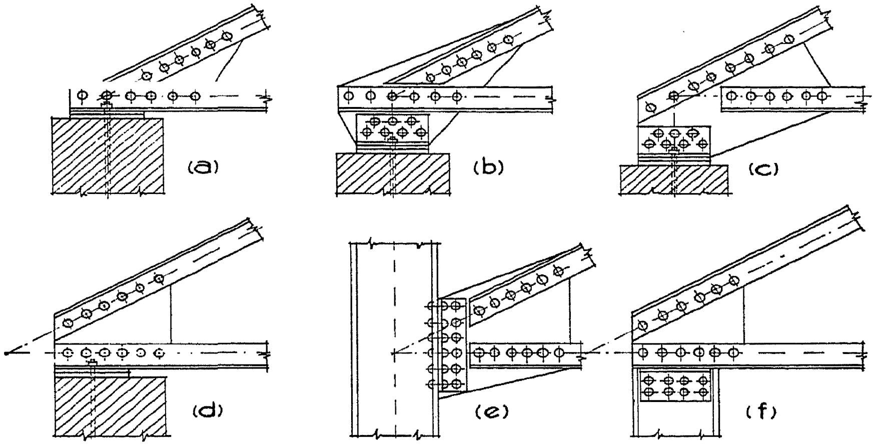


FIG. 100

In Fig. 100 (*d*) and (*f*) the working lines in the two chords and the center line of the support do not meet in a common point, thus producing eccentricity and therefore a bending moment in the connection. If possible, it is well to avoid this type of connection.

Fundamentally, the size of the bearing plates on a masonry wall depends upon the magnitude of the reaction and the allowable bearing stress of the masonry. It is considered good practice, however, to extend the end of the truss at least 6″ beyond the center line of the bearing to provide a minimum bearing length of 12″. To help to distribute the truss load more uniformly, a *sole plate* is riveted to the lower chord or shoe angles, the sole plate in turn resting upon the *bearing plate*. See Fig. 101. Both the sole plate and bearing plate should have minimum thicknesses of ½″. They should not project more than 3″ or 4″ beyond the sides of the angles and slotted holes should be provided for anchor bolts. Due to expansion and contraction, the slotted holes should be sufficiently long to provide for a movement of ⅛″ for every 10′0″ of truss span. Anchor bolts should never be less than ⅝″ in diameter, and the slotted holes through which they pass should be at least 5/16″ greater than the diameter of the bolts. When the span of the truss exceeds about 80′0″, a rocker or roller bearing should be employed.

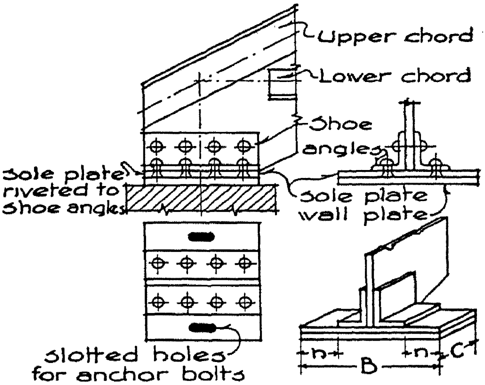

Upper chord
Lower chord
Shoe angles
Sole plate riveted to shoe angles
Sole plate
wall plate
slotted holes for anchor bolts
n
B
n
C

FIG. 101

123. **Bearing Plates.** As has been explained in the previous

article, the construction of a truss-bearing on a masonry wall consists of the angles of the lower chord or shoe angles, the sole plate and wall or bearing plate. The area of the plates is found by dividing the load due to the truss by the allowable bearing unit stress of the masonry. These allowable stresses for various kinds of masonry are given in Table XXI. Having determined the area of the plates, the thickness must be computed. The design procedure is similar to the design of bearing plates for beams. Referring to Fig. 101,

Table XXI. Allowable Bearing Strength of Masonry Walls

Stone concrete, 2,000#/in^2 ultimate	500#/in^2
Common brick, lime mortar	100 "
Common brick, lime-cement mortar	200 "
Common brick, cement mortar	250 "
Hard brick, cement mortar	300 "
Rubble, cement mortar	150 "
Rubble, lime-cement mortar	100 "

R = the reaction of the truss, in pounds
B = the length of the plates, in inches
C = the width of the plates, in inches
Area of the plates = $B \times C$ = R divided by the allowable bearing strength of the masonry
w = the actual bearing stress on the masonry, in pounds per square inch

$$n = \frac{B - (\text{distance between edges of outstanding legs of shoe angles})}{2} =$$

the projection of plates on each side of angles, in inches
f = the allowable bending stress of steel, usually 18,000#/in^2
t = the combined thickness of the sole and wall plates, in inches

To compute t, the required thickness, substitute the known values in the formula

$$2 \times f$$

To illustrate the design of a bearing on a wall support, assume the reaction from the truss to be 30,000#. The shoe angles are 5" x 3" x ½" short legs outstanding, and the gusset plate, to which they are attached, is ½" in thickness. The wall on which the truss is supported is built of common brick laid up with cement mortar.

The allowable bearing unit stress of the masonry is 250#/in², Table XXI, therefore the required area of the plates is 30,000 ÷ 250 or 120in². Accept a 12" x 12" plate, thus B and C are each 12". Then the actual bearing stress on the masonry will be $w = 30{,}000 \div 144$, or 209#/in².

The distance between the outer edges of the shoe angles is 3" + 3" + ½" or 6.5", therefore $n =$ 2.75".

Substituting in the formula

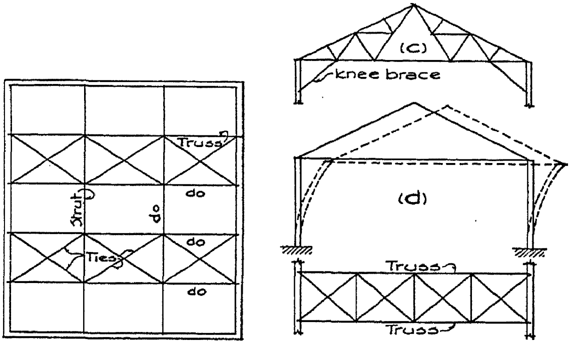
$$t = \frac{3 \times w \times B \times n}{2 \times f}$$

$$\frac{3 \times 209 \times 12 \times 2.75}{2 \times 18{,}000} \qquad 0.76''$$

This is the required thickness of the sole plate and wall plate combined. Since ½" is the minimum thickness, the sole plate and wall plate will each be 12" x 12" x ½", slotted to receive ⅝" anchor bolts.

124. **Bracing of Trusses.** Trusses are designed on the assumption that they are maintained in vertical planes. Wind loads on the sides and ends of a building, and vibration due to machinery, produce forces which must be counteracted by a system of bracing. The stresses developed in such bracing cannot be accurately estimated. Experience and judgment on the part of the designer are called upon in determining the size of the sections to be used. For average conditions, bracing is generally confined to the planes of the upper and lower chords as illustrated in Fig. 102 (*b*) and (*e*). It usually consists of ¾" diameter rods extending diagonally between the trusses. Generally the purlins and roof construction provide sufficient rigidity to brace the trusses in the planes of the upper chord. If a ceiling is provided, its construction helps in bracing the lower chord. Diagonal bracing may be used between all trusses or in alternate bays as indicated in Fig. 102 (*b*). In the latter case compression members should be used between trusses

as indicated. Such members are generally composed of two angles designed on the basis of $\frac{l}{r}$ not to exceed 200. This is the slenderness ratio, l being the length of the member in inches and r its least radius of gyration.



(b) Lower chord bracing (e) Upper chord bracing

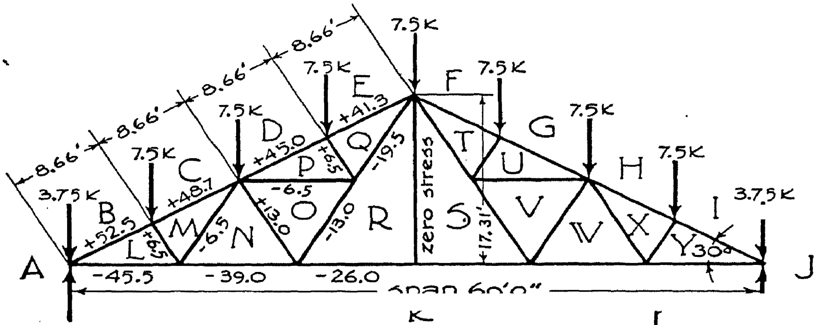
FIG. 102

If, for instance, the spacing of the trusses is 18′0″, $\frac{l}{r} = 200$ or $r = \frac{l}{200} = \frac{18 \times 12}{200} = 1.08''$, the required least radius of gyration. Referring to Table XVII, we find that a section composed of two angles $3\frac{1}{2}'' \times 2\frac{1}{2}'' \times \frac{1}{4}''$, with long legs back to back separated by $\frac{3}{8}''$, has a least radius of gyration of 1.09″ and therefore is acceptable.

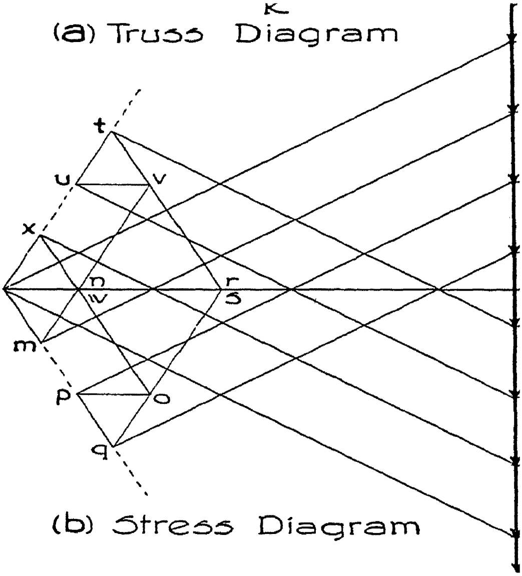
When trusses are supported by columns with no provision for transverse bracing, there is a tendency toward distortion due to wind loads as illustrated in Fig. 102 (*d*). It is imperative that bracing is added to overcome this tendency to rack. This is readily accomplished by the use of *knee braces*, shown in Fig. 102 (*c*). The stress developed in this type of bracing is found by the method explained in Art. 90. The column, of course, must be designed for an axial compressive load as well as for bending due to the force exerted by the brace.

125. **Design of a Steel Truss.** Let it be required to design a steel eight-panel Fink truss having a span of 60′0″ and a pitch of

30°. The trusses are spaced 17′0″ on centers. The roof construction consists of ⅜″ slate, 1″ sheathing, rafters 2″ x 6″—24″ o.c. and 10″ 15.3# channels for purlins. Use equivalent vertical loads for combined wind and snow.



(a) Truss Diagram



(b) Stress Diagram

FIG. 103

Computation of Loads. A truss diagram is first drawn, Fig. 103 (*a*), and the distance between panel points is found to be 8.66′. Since the trusses are 17′0″ apart, the number of square feet of roof surface exerting forces at each panel point is 17 × 8.66 or 147.22ft^2.

Computing the loads per square foot of roof surface,

⅜″ slate roofing	= 13#	Table II
1″ sheathing	= 3#	Table II
2″ x 6″ rafters, 24″ o.c.	= 1#	Table III
Equivalent vertical load for wind and snow combined	= 24#	Table VIII
Approximate weight of truss	= 8#	Table V
Total	= 49#/ft^2	

The load at each panel point, therefore, will be 49 × 147.22 = 7,213.78#, not including the weight of the purlins. Each purlin is 17′0″ in length and weighs 15.3 pounds per linear foot, therefore its weight is 17 × 15.3 or 260.1#. Hence 7,213.78 + 260.1 = 7,473.88#, the total vertical load at each interior panel point. Call this 7,500# or 7.5 kips. The panel loads at the ends will be ½ × 7.5 or 3.75 kips, and the total vertical load on one truss will be 7.5 × 8 or 60 kips. Each reaction will be 30 kips.

Determining the Stresses. This truss is symmetrically loaded and therefore the method of drawing the stress diagram explained in Art. 77 may be employed. If preferred, the method of introducing a substitute member, explained in Art. 76, may be used. Regardless of the method, the complete stress diagram is shown in Fig. 103 (*b*).

The lengths of the lines in the stress diagram are measured, thus establishing the magnitude of the stresses in the members. The character of the stresses are determined as explained in Art. 66. These stresses are noted on the truss diagram, Fig. 103 (*a*), and are also shown in Table XXII. It should be noted that the stresses have been found for this truss by the construction of a stress diagram. However, since there are no suspended loads and the truss is symmetrically loaded, the stresses might have been determined by the use of coefficients given in Table IX and explained in Art. 81. Refer to this table and verify the stresses. These tables are time savers but their use is limited since they are applicable only to symmetrically loaded trusses with vertical loads applied to the upper chords.

Selection of the Members. The design of compression and tension members for steel sections is explained in Arts. 110 and 111.

Table XXII. Stresses and Sections for Steel Fink Truss, Fig. 103

Member	Character of stress	Magnitude of stress in kips	Section
BL	Compression	52.5	2 ∟s 4 x 3 x 5/16
CM	Compression	48.7	2 ∟s 4 x 3 x 5/16
DP	Compression	45.0	2 ∟s 4 x 3 x 5/16
EQ	Compression	41.3	2 ∟s 4 x 3 x 5/16
LK	Tension	45.5	2 ∟s 3 x 2½ x 5/16
NK	Tension	39.0	2 ∟s 3 x 2½ x 5/16
RK	Tension	26.0	2 ∟s 2½ x 2 x ¼
LM	Compression	6.5	2 ∟s 2½ x 2 x ¼
NO	Compression	13.0	2 ∟s 3½ x 2½ x ¼
PQ	Compression	6.5	2 ∟s 2½ x 2 x ¼
MN	Tension	6.5	2 ∟s 2½ x 2 x ¼
OP	Tension	6.5	2 ∟s 2½ x 2 x ¼
OR	Tension	13.0	2 ∟s 2½ x 2 x ¼
QR	Tension	19.5	2 ∟s 2½ x 2 x ¼
RS	No stress	No stress	1 ∟ 2½ x 2 x ¼

The compression members may be readily determined by the use of Table XVII. Table XVIII permits the selection of members resisting tensile forces. Two 2½″ x 2″ x ¼″ angles, long legs back to back will be used as the minimum section with the exception of member *RS*. The purpose of this member is to prevent sag in the lower chord and a single 2½″ x 2″ x ¼″ angle will be employed.

The upper chord will be made continuous from the support to the ridge. The maximum stress in this member occurs in the section adjacent to the support, *BL*. Its magnitude is 52.5 kips, its length is 8.66′ and its character is compression. Referring to Table XVII we find that two 4″ x 3″ x 5/16″ angles, having a length of 9′0″, will safely support a force of 54 kips and therefore is acceptable. Observe that the heavy vertical line in the table occurs between the 12′ and 14′ lengths, showing that 12′ is the limiting length for this section if the slenderness ratio, $\frac{l}{r}$, is to be within the limit of 120. Note also that safe loads in the table are given with respect to both major axes, *X*—*X* and *Y*—*Y*, the smaller value, determined by the least radius of gyration, being the allowable safe load. This table is computed for angles spaced 3/8″ apart, the thickness of the gusset plate generally used. Since the section selected is adequate for member *BL*, it will be sufficiently large for *CM*, *DP* and *EQ* and will be made continuous, thus simplifying the fabrication.

Member *NO* is 10′0″ in length and resists a compressive force of 13 kips. Table XVII shows that a section composed of two 3½″ x 2½″ x ¼″ angles, 10′0″ in length, will resist a safe load of 31 kips (*Y*—*Y* axis), and therefore is acceptable. In this instance the section is determined by the limiting slenderness ratio of 120. Any smaller section shown in the table has a slenderness ratio exceeding 120 and therefore is inadequate.

Members *LM* and *PQ* each resist compressive forces of 6.5 kips and have lengths of 5′0″. Table XVII shows that the minimum section, two 2½″ x 2″ x ¼″ angles will be acceptable for these members.

The members *LK* and *NK* will be made of one continuous member. These two members are in tension, *LK* having a stress of 45.5 kips and *NK* a stress of 39.0 kips. Since *LK* is the greater, its stress will determine the size of the member. We will assume that ¾″ rivets will be used throughout the truss and that the allowable tensile unit stress of structural members will be 18,000#/in^2. As each member of the truss is to consist of two angles and the member *LK* is to resist a force of 45.5 kips, each angle of the member *LK* must resist a force of 45.5 × ½ or 22.75 kips. Referring to Table XVIII we find that a 3″ x 2½″ x ⁵⁄₁₆″ angle, punched for a ¾″ rivet, will safely resist a tensile force of 24.3 kips, therefore a section composed of two 3″ x 2½″ x ⁵⁄₁₆″ angles is acceptable for the members *LK* and *NK*. It should be noted that the length of a member in direct tension is not a factor in its design.

The member *RS* has no stress, consequently *RK* and *SK* have equal tensile stresses, 26 kips each. These two members will be made continuous. As the member will be composed of two angles, 26 × ½ = 13 kips. A single 2½″ x 2″ x ¼″ angle will resist a tensile force of 15.1 kips, Table XVIII, and therefore members *RK* and *SK* will consist of two 2½″ x 2″ x ¼″ angles, the minimum section.

Members *OR* and *QR* will also be made continuous. The tensile stress in *QR*, 19.5 kips, being the greater, will govern the design. Each angle must resist a force of 19.5 × ½ = 9.75 kips. The minimum size angle, 2½″ x 2″ x ¼″, has a safe tension value of 15.1 kips, Table XVIII, hence members *OR* and *QR* will be continuous, composed of two 2½″ x 2″ x ¼″ angles.

Members *MN* and *OP* each have tensile stresses of 6.5 kips and Table XVIII shows that the minimum section, two $2\frac{1}{2}'' \times 2'' \times \frac{1}{4}''$ angles, is acceptable.

The truss being symmetrical, the members similarly located in each side of the truss will consist of like sections. The relative positions of the various members are shown in Fig. 104.

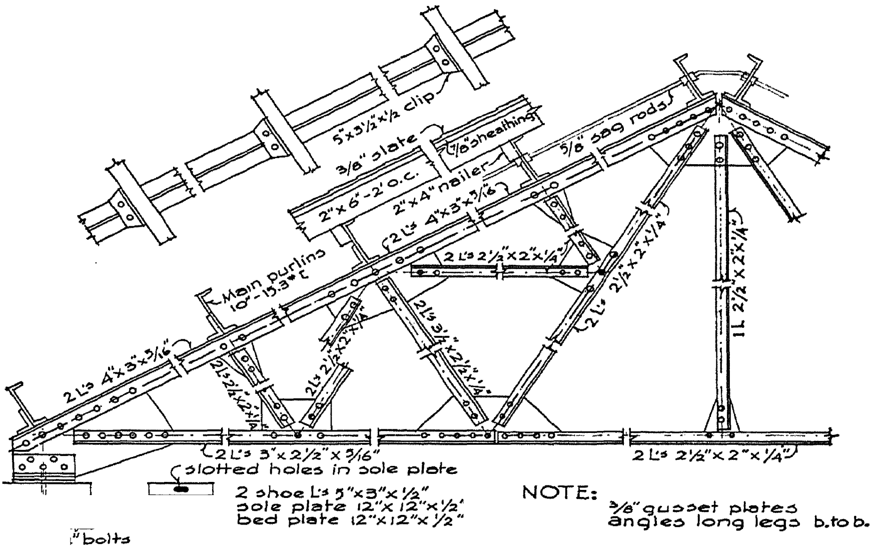


FIG. 104

Number of Rivets. In determining the number of rivets to use at the joints, let us assume them to be $\frac{3}{4}''$ in diameter. The gusset plates will be $\frac{3}{8}''$ in thickness. A minimum number of two rivets will be used in the connection of each member to the gusset, and the allowable working stresses for power driven rivets will be

Shearing unit stress	13,500#/in^2
Single-bearing unit stress	24,000#/in^2
Double-bearing unit stress	30,000#/in^2

These unit stresses have been used in computing the allowable working values in Table XIX, consequently this table may be used directly in determining the controlling values as explained in Arts. 112 and 113.

Let us consider first the member *BL* where it joins the gusset plate at the heel joint. The rivets are $\frac{3}{4}''$ diameter, the thickness of the angles is $\frac{5}{16}''$ and the gusset is $\frac{3}{8}''$. Referring to Table XIX we find the following working values for *one* rivet:

$\frac{3}{4}''$ rivet, double shear = 11,930#
$\frac{3}{8}''$ gusset, double bearing = 8,440#, controlling value
two $\frac{5}{16}''$ angle legs, single bearing, 2 × 5,630 = 11,260#

8,440#, double bearing, being the smallest of these three magnitudes, is therefore the controlling value of one rivet. Since the force to be transferred from the member to the gusset is 52.5 kips, 52.5 ÷ 8.44 = 6+. Therefore we will use seven rivets in the connection as shown in Fig. 104.

The member *LK* transfers a force of 45.5 kips to this same gusset plate. Since the angles of this member are also $\frac{5}{16}''$ in thickness, 8,440# is again the controlling value of one rivet. Therefore 45.5 ÷ 8.44 = 5+, hence we will use six rivets in member *LK*.

The total load on the truss is 8 × 7.5 or 60 kips, hence the load transferred from the truss to each bearing wall is 60 × $\frac{1}{2}$ or 30 kips. The design of bearing plates is illustrated in Art. 123. For this truss the shoe angles will be $\frac{1}{2}''$ thick and, since the gusset plate is $\frac{3}{8}''$ in thickness, 8,440# is the controlling value of one rivet. Therefore 30 ÷ 8.44 = 3+; therefore, theoretically, four rivets are required in the shoe angles. For symmetry five rivets are used, staggered as indicated in Fig. 104.

Consider next the upper chord at the joint *BCML*. As explained in Art. 114, the force to be transferred to the gusset plate, since the upper chord is continuous, is 52.5 − 48.7 or 3.8 kips. Again the controlling value of one rivet is 8,440#, therefore 3.8 ÷ 8.44 gives, theoretically, less than one rivet as the required number. Two rivets will be used as a minimum number at any connection. Where purlins occur, there should be a sufficient number of rivets to develop the load transferred to the truss. In this case the purlin load is 7,500# and since the controlling value of one rivet is 8,440#, it is evident that two rivets are ample.

In a similar manner, two, the minimum number of rivets, would be sufficient for the forces to be transferred at the joint *CDPONM*. However, owing to the required dimensions of the gusset plate, five rivets will be used in the upper chord as shown in Fig. 104.

Observe that the upper chord terminates at the joint *EFTSRQ*, hence the full stress must be developed by the rivets. The stress in the member *EQ* is 41.3 kips and, as the controlling value of one rivet is 8,440#, $41.3 \div 8.44 = 4+$, hence five rivets will be used in member EQ.

The procedure just explained is used to determine the number of rivets for the remaining joints of the truss. It should be remembered that at least two rivets are used for the connection of each member to the gusset plate and that often more than the required number are used if the dimensions of the gusset are sufficiently large.

It is not to be supposed that the architect or structural engineer is always required to design the details of a truss. That is generally done by the fabricating company. The designer, of course, determines the size of the members and, before the truss is constructed, the fabricating company submits shop-drawings for approval. These shop-drawings give complete details, length and sections of members and gusset plates as well as exact rivet spacing. Obviously, if the designer is familiar with the basic principles of structural detailing, he may design more efficiently and is less likely to call for uneconomical sizes or awkward connections.

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